# Impact of Own Brand Product Introduction on Optimal Pricing 

# Models for Platform and Incumbent Sellers 

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#### Abstract

Sales on the e-commerce platform in the United States have experienced explosive growth and are projected to surpass 740 billion in 2023. The expansion of the platform's traditional role as a reseller into an online marketplace and the introduction of its own brand products have stoked a huge fear among the incumbent sellers. The platform's unfair anti-competitive practice further aggravates the situation. Consequently, politicians and regulators have proposed prohibiting platforms from introducing own brand products in order to protect the incumbent sellers. This study addresses two questions of critical interest to both the policy makers and the incumbent sellers. First, how does the platform's introducing its own brand product affect the incumbent sellers? Second, how effective is the proposed policy in terms of protecting the incumbent sellers? We examine the impact of the platform's own brand introduction on the incumbent sellers under two prevailing sell-on and sell-to pricing contracts. We find that the proposed legislation "that prohibits platforms from both offering a marketplace for commerce and participating in that marketplace" does not have the desired outcome of helping the incumbent sellers. Instead, it forces the platform to adopt only the sell-to contract with the own brand introduction that hurts the sellers under most market conditions. Interestingly, when the own brand introduction is banned under the sell-to contract, the incumbent sellers can be better off because the platform's strategic reaction to the enforcement can lead to the best scenario for the incumbent sellers. If the ban is imposed on both the sell-on and sell-to contracts, the platform's best response is to add another new brand competing with the incumbent sellers, which can also help the incumbent sellers, however, not as much as in the case of the enforcement only under the sell-to contract.


Keywords: own brand product; sell-on contract; sell-to contract; e-commerce platform; antitrust regulation
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## 1 Introduction

The recent huge success of e-commerce platforms' own brand products has generated widespread attention from the general public, politicians, and regulators. On March 8, 2019, U.S. Senator Elizabeth Warren, one of the front runners of Democratic presidential candidates at the time, argued for a stronger enforcement of antitrust law over the tech behemoth Amazon for aggressively selling its own brand products. She further proposed "legislation that prohibits platforms from both offering a marketplace for commerce and participating in that marketplace" at the same rally in Long Island, New York. On July 17, 2019, the European Union announced a plan to investigate whether Amazon was misusing its dual role as both the marketplace for independent sellers and the platform of its own products (The New York Times, 2019). The regulatory agencies from both the United State and the European Union share the same concerns over the phenomenal growth of e-commerce platforms' own brand products as it could negatively impact the independent sellers.

The concerns of the U.S. and European regulators are not unfounded. For example, Amazon sells more than 70 own brand products from clothes to baby wipes, including the recent launch of over 50 own brand fashion labels for men, women and kids (Business Insider, 2018). There is an explosive growth for the number of Amazon's own brands and the number of products within each product category. During 2017-2018, the number of own brands increases by at least $200 \%$ and the number of own-brand products increases by at least $400 \%$. According to Robinson Humphrey of SunTrust (Business Insider, 2017a), Amazon's own brand product sales are expected to grow at a fast pace over the next five years and post $\$ 31$ billion in sales by 2022, more than Macy's Inc.'s annual revenue of 2018. Other e-commerce platforms (e.g., Jet.com, Overstock.com, etc.) have made the similar move of introducing their own brand products. Figure 1 shows examples of Amazon's own brand products (e.g., AmazonBasics and Pinzon by Amazon) next to competing products sold by the third-party sellers.

In addition to introducing own brand products, e-commerce platforms are found to leverage their marketplace infrastructure to influence consumers in favor of their own brand products over those of the competitors (Business Insider, 2017a). According to The Capitol Forum (2016), Amazon prioritizes its own brand products over the other products by using its platform in a biased manner.


Figure 1：Examples of Amazon＇s private－label products（e．g．，AmazonBasics and Pinzon by Ama－ zon）and competing products

The analysis reveals that Amazon gives its own brands in the＂Customers Who Bought This Item Also Bought＂promotional carousel and restricts competitors＇access to this prominent placement on product pages so as to direct consumers to its own brand products．It has been claimed that Amazon＇s own brand takes the top position in product pages，recommendations，and search results， receiving increased awareness and perceived product quality（The Capital Forum，2016；Thomson and Hansen，2016）．Moreover，since Amazon has built up the credibility in customer support over the years as the company with the best customer service reputation（The Harris Poll，2018； Business Insider，2017b），customers＇familiar experiences in Amazon＇s customer support，ease of checkout and return all lead to a better perception for Amazon＇s brands than non－Amazon brands． Consequently，Amazon＇s growing own brand business stokes＂huge fear＂in the independent sellers （CNBC，2018）．

There is an urgent need for both the independent sellers and the policy makers to better under－ stand the impact of e－commerce platform＇s own brand products．For the sellers，selling products on Amazon is essential because of its well－developed infrastructure（Forbes，2018）．In other words， the absence in Amazon is not an option for sellers（Thomson and Hansen，2016）．The number of active Amazon customer accounts worldwide in the first quarter of 2016 is 310 million and more than 300,000 small－and mid－sized sellers started selling on Amazon marketplace in 2017 （Internet Retailer Research，2018）．For the policy makers，given the fact that independent sellers have few op－
tions when the e-commerce platforms introduce own brand products, it is important to understand how best to protect the affected sellers.

From the policy perspective, two issues are of particular interest. First, the policy makers would like to comprehensively understand the effect of e-commerce platform's introduction of the own brand product on the incumbent sellers. The answer to this issue unfortunately cannot be inferred from the extant literature (e.g., Nasser et al., 2013; Kumar and Steenkamp, 2007; Raju et al., 1995) due to the uniqueness of e-commerce platforms. E-commerce platforms have expanded their traditional roles as resellers into online marketplaces (e.g., Hao and Fan, 2014), where sellers directly sell products on the marketplace and use agency pricing contract (hereafter termed the sell-on contract) instead of traditional wholesale contract (hereafter termed the sell-to contract). To illustrate, sellers in the U.S. Amazon marketplace offer over 350 million products, accounting for sixty five percent of total sales in Amazon (Internet Retailer Research, 2017). Furthermore, the traditional view of the reseller's own brand being lower-grade is no longer valid for the e-commerce platform (e.g., Amazon). Rather, the e-commerce platform can take advantage of its power as the marketplace operator to sway consumers to become more favorable to its own brand. To address the first issue, we develop a game-theoretic model and determine the equilibrium of the cases with and without own brand products in both the sell-to (i.e., the traditional wholesale contract) and sell-on (i.e., the agency pricing contract) scenarios in Section 4. The results indicate that how the platform's own brand introduction affects the incumbent sellers critically depends on pricing contracts between the incumbents and the platform.

The second issue of interest to the policy makers is to ascertain the effectiveness of proposed policies aimed at mitigating the threat from the platform's own brand to incumbent sellers. One popular proposal, such as Senator Warren's, is to prohibit Amazon's own brand products in the Amazon marketplace. Understanding Amazon's strategic response is essential to assess the ultimate effectiveness of such policy. To address this issue, in Section 5, we examine the e-commerce platform's incentive and preference for introducing own brand, and evaluate the effectiveness of three possible policies aimed at protecting the incumbent sellers.

Motivated by the urgent call on understanding the effect of e-commerce platform's introduction
of own brand products and the efficacy of proposed policies to curtail its impact on incumbent sellers, this study provides comprehensive step-by-step analyses of these critical issues. We first examine the current reality where the platform (e.g., Amazon) has introduced its own brand and there is no regulation in place, and describe the impact of own brand introduction on incumbent sellers under two prevailing types of pricing contracts. We next analyze the e-commerce platform's incentives in terms of whether to introduce its own brand and its preference for the type of pricing contract, and the corresponding effect of the platform's entry and contract decisions. Finally, we assess the efficacy of three possible policy options of prohibiting the platform from introducing its own brand by considering the platform's strategic move in response to the regulation.

We model two competing incumbent sellers who sell products in an e-commerce platform's marketplace. If the platform launches its own brand, the incumbents face an additional competitor in the marketplace where the platform's own brand can be (i) weaker than or equivalent to the incumbents' products, or (ii) stronger than the incumbents' products because of the favorable treatments in product pages positioning, recommendations, etc. The incumbents sell either directly on the marketplace (i.e., sell-on contract) or to the platform (i.e., sell-to contract). The fundamental difference between these two contracts is the sellers' control over the retail price: the retail price of the sellers' products is determined by each seller in the former contract but by the platform in the latter. In the sell-on contract, the incumbent sellers pay a portion of their revenue while they compete with the platform's brand, and the double marginalization issue disappears, which commonly exists across the wholesale contract and its variations. Hence, both the cooperative and competitive nature co-exist under the sell-on contract. In contrast, double marginalization dominates the outcome under the sell-to contract. We find that the impact of the e-commerce platform's own brand introduction differs across the contracts. Specifically, our result shows that in the absence of regulation, introducing the own brand product will not harm the incumbent sellers if the own brand is very weak (vs. incumbent sellers). Moreover, the incumbent sellers always profit more (less) when they compete with the platform's own brand than with another new brand under the sell-on (sell-to) contract, an important and new finding to the literature. However, as long as the own brand is not too weak, the platform's decisions on introducing its own brand and its choices
on pricing contracts always hurt the incumbent sellers.
To assess whether the proposed policy of prohibiting e-commerce platforms from introducing own brand products is effective, we analyze various policy options where the platform's own brand introduction is banned. We find that if the prohibition policy is implemented only under the sellto contract, the incumbent sellers can be better off because the e-commerce platform's strategic reaction to the enforcement can lead to the sellers' best-case scenario. In contrast, if the ban is enforced only for the sell-on contract, it can result in the worst scenario for the incumbents, resulting in profit losses in most market conditions. If the platform is banned from introducing its own brand under both contracts (i.e., an all-out enforcement), the platform's best response is to add another new brand competing with the incumbents, which can help the sellers avoid their worst scenario and make them profitable. Our research provides important and useful implications to the policy makers. In particular, we find that the platform's own brand introduction does not always hurt the incumbents sellers and more importantly, the proposed regulation does not necessarily help the incumbents. The impact of the policy critically depends on the types of contracts for the incumbent sellers, the relative advantage of the platform's brand to those of the incumbents, the price-sensitivity of the incumbents' market, and the commission rate paid to the platform. In summary, we have the following key findings. First, the impact of additional competition from the own brand to the incumbent sellers is the same across different contract types. Second, contrary to the politicians' original intention, the proposed policy is more effective to better protect the incumbent sellers under the sell-to contract rather than the sell-on contract. Third, an all-out ban of own brands for both contracts will result in the platform introducing a new brand, an outcome that could be helpful for incumbent sellers but not as effective as the ban only under the sell-to contract.

The rest of the paper is organized as follows. We review relevant literature in the next section. Then, we describe our model in Section 3. The equilibrium outcomes and main findings are presented in Sections 4 and 5. In Section 4, we examine the platform's incentive of its own brand introduction and discuss the impact on incumbent sellers under different market characteristics and pricing contract alternatives. We then explore the impact of various enforcement policies against
the platform's own brand introduction and the platform's strategic reaction in Section 5. In Section 6 , we present many extensions to our model, and conclude the paper in Section 7.

## 2 Literature Review

This paper contributes to the stream of research on e-commerce pricing contracts. Prior studies have analyzed strategic pricing decisions in online platforms (e.g., Huang et al., 2020; Sun et al., 2017; Zhu and Liu, 2018; Wu et al., 2015; Chen and Guo, 2014), particularly the agency pricing model that is common in online marketplaces (e.g., Tan et al., 2016; Abhishek et al., 2016; Hagiu and Wright, 2015; Hao and Fan, 2014; Jiang et al., 2011). Jiang et al. (2011) examine an e-commerce platform's cherry picking behavior as the platform learns the sellers' demands and Hao and Fan (2014) study pricing contracts in the e-book industry that has adopted the agency pricing model. Abhishek et al. (2016) analyze pricing contracts of competing retailers as the retailers extend their channels. Hagiu and Wright (2015) study the intermediary's different pricing contracts by focusing on the control of marketing decisions. A few studies consider the channel coordination issue in the online marketplaces (e.g., Zhu and Liu, 2018; Chen and Guo, 2014; Mantin et al., 2014).

Contributing to the aforementioned stream of research, we focus on the platform's incentive to introduce the own brand product in its own marketplace, and examine the effectiveness of the regulators' proposed ban on the platform (e.g., Amazon) from selling own brand products in its own marketplace. Our results show that the concern over the platform's introducing own brand products hurting the incumbent sellers is not unfounded, especially when the platform can decide when to introduce its own brand and which pricing contract to adopt. However, the impact is different across pricing contracts in terms of how the own or the new brand affects the incumbent sellers. The proposed policy does not always help the incumbent sellers as the policy makers fail to take the platform's strategic response to the regulation into consideration.

The literature of the channel coordination analyzes the order quantity and price (e.g., McGuire and Staelin, 1983; Coughlan, 1985; Moorthy, 1988), service (e.g., Boyaci and Gallego, 2004), process innovation (e.g., Gupta and Loulou, 1998), and contract design (e.g., Cachon and Kök, 2010). The retailer's penetration in the product market is widely documented in this stream of literature, and
the focus is on the retailer's positioning (e.g., Sayman et al., 2002), pricing (e.g., Chintagunta et al., 2002; Dhar and Hoch, 1997), and negotiation power (e.g., Morton and Zettelmeyer, 2004), as well as the national brand's strategic response to retailer's penetration (e.g., Nasser et al., 2013). These existing studies make the same assumption that the platform only acts as traditional re-seller and the brand seller's product is of better quality and analyze the own brand retailer's threat to the seller (e.g., Nasser et al., 2013; Kumar and Steenkamp, 2007; Mills, 1999; Chintagunta et al., 2002; Sayman et al., 2002; Dhar and Hoch, 1997).

While the sell-to contract (i.e., the typical wholesale contract) is widely studied, the sell-on contract, known as the agency model, in the context of the platform introducing own brand products has not been examined in the literature. There exist distinct differences between the sell-to and the sell-on contracts in terms of the existence versus absence of double marginalization, and who has the control over retail pricing. In this study, we examine the platform's own brand introduction under both contracts. We further contribute to the literature by examining whether the recent proposed policy of prohibiting the platform from introducing own brand products in its own marketplace is effective in helping the incumbent sellers under the sell-to and the sell-on contracts.

## 3 Model

Without loss of generality, we consider an established e-commerce platform and two incumbent sellers. We denote the seller that sells product $i$ seller $i, i \in\{1,2\}$. The sellers compete in the same product category in the platform's online marketplace. The platform has an option to introduce its own brand product $r$, and if so, three products, products $i$ 's and $r$, compete in the same category by serving a common set of consumers. For the sellers, there are two pricing contracts in the marketplace: selling on the platform's marketplace (the sell-on contract) or selling to the platform (the sell-to contract). In the former, the seller sets the retail price $p_{i}$ of product $i$ to sell to consumers and pays a commission on each item sold on the platform's marketplace. ${ }^{1}$ In the latter, as in the traditional wholesale pricing contract, the platform buys product $i$ from seller $i$ and sets the retail

[^0]price to sell to consumers. Under both contracts with the seller, the platform decides whether to sell its own product in the same category.

We first describe the demand functions. We use a superscript $s$ to denote the benchmark sellersonly scenario in which two incumbent sellers compete in the platform's marketplace. In the absence of the platform's product $r$, the benchmark scenario has the demand structure as follows:

$$
\begin{align*}
& D_{1}^{s}=\frac{1}{2}\left[1-p_{1}^{s}+\theta\left(p_{2}^{s}-p_{1}^{s}\right)\right]  \tag{1}\\
& D_{2}^{s}=\frac{1}{2}\left[1-p_{2}^{s}+\theta\left(p_{1}^{s}-p_{2}^{s}\right)\right]
\end{align*}
$$

where $p_{i}^{s}$ is retail price of product $i, i \in\{1,2\}$, and $\theta, \theta \in[0,1)$, is the degree of cross price sensitivity between the two incumbents. The parameter $\theta$ captures the intensity of competition between the incumbent sellers. The base level demands of both products are normalized to 1. If the platform decides to sell its own brand product in the same market, we use a superscript o to denote the own brand scenario. In the presence of the platform's own brand product, the demand is

$$
\begin{align*}
& D_{r}^{o}=\frac{1}{2+a}\left[a-p_{r}^{o}+\frac{1}{2}\left[\delta\left(p_{1}^{o}-p_{r}^{o}\right)+\delta\left(p_{2}^{o}-p_{r}^{o}\right)\right]\right] \\
& D_{1}^{o}=\frac{1}{2+a}\left[1-p_{1}^{o}+\frac{1}{2}\left[\theta\left(p_{2}^{o}-p_{1}^{o}\right)+\delta\left(p_{r}^{o}-p_{1}^{o}\right)\right]\right]  \tag{2}\\
& D_{2}^{o}=\frac{1}{2+a}\left[1-p_{2}^{o}+\frac{1}{2}\left[\theta\left(p_{1}^{o}-p_{2}^{o}\right)+\delta\left(p_{r}^{o}-p_{2}^{o}\right)\right]\right]
\end{align*}
$$

where $p_{i}^{o}$ and $p_{r}^{o}$ are the retail prices of product $i, i \in\{1,2\}$ and product $r$, respectively. The parameter $\delta \in[0,1)$ is the degree of cross price sensitivity between the incumbent $i$ 's and the platform's product, representing the intensity of competition among them. The base level demands for platform's own brand $r$ and incumbent sellers' products $i$ are $a$, and 1 respectively. The price differentials are divided by 2 since each product competes with two others.

The intercept terms $\frac{a}{2+a}$ and $\frac{1}{2+a}$ capture the respective strength of platform's own brand and incumbent sellers' brand (Sayman and Raju, 2004). In the analyses throughout this paper, we adopt a general assumption that $a \geq 0$ in Equation (2) to cover all possible base level demands of the platform's own brand. Specifically, it captures the situation where the platform's own brand is weaker than or equivalent to the incumbent sellers' products (i.e., $a \leq 1$ ), as well as the situation where platform's own brand is stronger than the incumbent sellers' products (i.e., $a>1$ ). The
former reflects the situation where the platform is not able to build a stronger brand than the incumbent sellers, and the latter reflects the recent observation that platform's own brand has unfair competitive advantages over other products in the platform's own marketplace. For instance, Amazon has a higher brand reputation, credibility in product support, and marketing advantages than other sellers. More importantly, Amazon brand most likely takes the top position in the product pages, recommendations, and search results, which improves its perceived quality relative to other brands' (Thomson and Hansen, 2016; The Capital Forum, 2016). The demand functions in our model are widely used in the literature involving multi-product competition (e.g., Lee and Staelin, 2000; Sayman and Raju, 2004; Raju et al., 1995). This demand system is adapted from Shubik and Levitan (1980), and it is consistent with utility maximizing consumers with quadratic utility functions (see Raju et al., 1995). Apart from being consistent with the previous literature, employing the above structure helps us capture both consumers' preference on different brands (i.e., product $i$ and $r, i \in\{1,2\})$ and the competition intensity among different products represented by the degrees of cross price sensitivity parameters $\delta$ and $\theta$.

The sequence of events are as follows. Under the sell-on contract in the benchmark case (scenario $s)$, the incumbent sellers simultaneously set retail prices of product $i\left(p_{i}, i \in\{1,2\}\right)$ in stage 1 . In stage 2, demand is realized and the sellers pay the commissions to the platform. Under the sell-to contract, in stage 1 , the sellers set wholesale price $w_{i}, i \in\{1,2\}$. In stage 2 , given the wholesale prices, the platform sets retail prices of the products, $p_{i}, i \in\{1,2\}$. Demand is realized in stage 3 .

In the presence of the platform's own brand (scenario o), the platform's retail price decision for its product, $p_{r}$, is made simultaneously with retail prices of other two products in each contract. In Section 5, we examine the platform's preferences for whether to launch its own brand product and for contract alternatives. We assume that the platform and sellers source their products at a fixed per-unit cost. ${ }^{2}$ All players are risk neutral and have common set of information before making decisions.

In the next section, we examine whether the platform's introduction of its own brand product

[^1]hurts the incumbent sellers under the sell-on and sell-to contracts. Specifically, we compare the incumbent sellers' profits before and after the introduction of platform's own brand product and discuss the impact of the platform's own brand introduction on the incumbent sellers.

## 4 Platform's Introduction of Its Own Brand

In this section, we examine the impact of platform's introduction of own brand on the incumbent sellers, and assess the effectiveness of the proposed policy of prohibiting the dual role of both operating and participating in the marketplace. We first derive the subgame perfect equilibria using backward induction under the sell-on and sell-to contracts respectively.

### 4.1 Platform's Own Brand Introduction under Sell-On Contract

We first derive the equilibrium outcome of the benchmark scenario $s$, where the two incumbent sellers compete in the platform's marketplace. Under the sell-on contract, in stage 1 of the game, given the commission rate $\alpha$, two sellers determine their own optimal retail prices, $p_{i}^{s}, i \in\{1,2\}$, simultaneously to maximize their profits:

$$
\begin{equation*}
\max _{p_{i}^{s}} \pi_{i}^{s}=(1-\alpha) p_{i}^{s} D_{i}^{s} \tag{3}
\end{equation*}
$$

where $D_{i}^{s}, i \in\{1,2\}$, is defined in Equation (1). The profit of the platform is $\pi_{r}^{s}=\alpha\left(p_{1}^{s} D_{1}^{s}+p_{2}^{s} D_{2}^{s}\right)$. Solving the first-order conditions, we obtain the optimal retail price of product $i$. Then, we derive the equilibrium demands of each product and profits for the platform and incumbent sellers.

Next, we derive the equilibrium outcome when the platform introduces its own brand to compete with the incumbent products (scenario o). Under the sell-on contract, in stage 1 of the game, both the platform and sellers solve for the optimal retail prices to maximize their profits given the commission rate $\alpha$ :

$$
\begin{align*}
\max _{p_{r}^{o}} \pi_{r}^{o} & =p_{r}^{o} D_{r}^{o}+\alpha\left(p_{1}^{o} D_{1}^{o}+p_{2}^{o} D_{2}^{o}\right)  \tag{4}\\
\max _{p_{i}^{o}} \pi_{i}^{o} & =(1-\alpha) p_{i}^{o} D_{i}^{o} \tag{5}
\end{align*}
$$

where $i \in\{1,2\}$, and $D_{r}^{o}$ and $D_{i}^{o}$ are specified in Equation (2). Similarly, solving the first-order conditions of the platform and the sellers simultaneously, we derive the retail prices. We then derive the equilibrium demands and profits. We assume that the commission charged to the incumbent sellers is between 0 and $\frac{1}{2}$ as in the practice. Detailed derivation and all equilibrium outcome are delegated to the online appendix.

Under the sell-on contract, we summarize the impact of the platform's own brand introduction on the incumbent sellers by comparing the sellers' profits between the benchmark and the own brand scenarios in the following proposition.

Proposition 1. Under the sell-on contract, introduction of the platform's own brand doesn't always hurt the incumbent sellers. The incumbent sellers can be better off, i.e., $\pi_{i}^{o}>\pi_{i}^{s}, i \in\{1,2\}$, if and only if (i) $a<a_{11}$ and $\delta<\delta_{1}$ or (ii) $a>a_{12}$.

Proof. All proofs and threshold values are in the online appendix unless indicated otherwise.

Surprisingly, Proposition 1 shows that the incumbent sellers can be strictly better off in the presence of the platform's own brand introduction. The sellers can profit more when the own brand product has highly weak (as in Part (i) of Proposition 1) or strong (as in Part (ii) of Proposition 1) brand advantage compared to their own products.

Under the sell-on contract, the platform takes a portion of the sellers' revenue while at the same time it competes against the sellers with its own brand. The platform maximizes its profit by juggling these two sources of profit. The own-brand introduction benefits the incumbents only when the competition with the platform does not encroach the incumbent sellers' market too significantly. If the base level demand of the platform's product is close to the sellers' (i.e., $a$ is approaching 1 ), this win-win situation will not materialize since the own brand platform cannot increase its profit without trouncing its opponents. The incumbents also respond to the new entrant. For condition (i), when $a$ is low, the incumbents' prices can be set relatively high to the platform's; the prices can be even higher with the own brand than without it. This is more likely to benefit the incumbents when $\delta$ is low where competition intensity with the own brand is low. For condition (ii), as $a$ increases to a certain level, the platform increases its own brand price and thus its competing incumbents
can also adjust their prices accordingly. When $a$ becomes high enough, the platform's brand par excellence can help the sellers largely alleviate its head-on competition against the platform. The platform increases its retail price as its brand performance is higher represented by a higher $a$, which in turn allows the incumbent sellers to set higher prices and increase their profits.

### 4.2 Platform's Own Brand Introduction under Sell-To Contract

We now explore the impact of platform's own brand product introduction under the sell-to (i.e., traditional wholesale) contract. Similar to our analyses in the sell-on contract, we first derive the equilibrium outcome of the benchmark scenario where the two incumbent sellers compete in the absence of the platform's own brand product. In stage 2 of the game, given the wholesale prices $w_{i}^{s}$, $i \in\{1,2\}$ from the incumbent sellers, the platform determines optimal retail prices for each product $p_{i}^{s}, i \in\{1,2\}$ to maximize its profit:

$$
\begin{equation*}
\max _{p_{1}^{s}, p_{2}^{s}} \pi_{r}^{s}=\left(p_{1}^{s}-w_{1}^{s}\right) D_{1}^{s}+\left(p_{2}^{s}-w_{2}^{s}\right) D_{2}^{s} \tag{6}
\end{equation*}
$$

where $D_{i}^{s}, i \in\{1,2\}$, is found in Equation (1). Solving the platform's first-order conditions, we derive the retail price of product $i, p_{i}^{s}$, as a function of the wholesale prices $w_{i}^{s}, i \in\{1,2\}$. In stage 1 of the game, anticipating the platform's best reaction functions, the two sellers derive their optimal wholesale prices by solving their respective first-order conditions: for $i=\{1,2\}$,

$$
\begin{equation*}
\max _{w_{i}^{s}} \pi_{i}^{s}=w_{i}^{s} D_{i}^{s} \tag{7}
\end{equation*}
$$

Substituting the optimal wholesale prices to the platform's retail pricing functions, we obtain the optimal retail prices. Then, we derive the equilibrium demand for each product and the profits for the platform and the incumbent sellers.

We next derive the equilibrium outcome when the e-commerce platform introduces its own brand product. In stage 2 of the game, the platform determines optimal retail prices for all three products
to maximize its profit:

$$
\begin{equation*}
\max _{p_{r}^{o}, p_{1}^{,}, p_{2}^{o}} \pi_{r}^{o}=p_{r}^{o} D_{r}^{o}+\left(p_{1}^{o}-w_{1}^{o}\right) D_{1}^{o}+\left(p_{2}^{o}-w_{2}^{o}\right) D_{2}^{o} \tag{8}
\end{equation*}
$$

where $D_{r}^{o}, D_{1}^{o}$ and $D_{2}^{o}$ are specified in Equation (2). In stage 1 of the game, two sellers determine their own optimal wholesale prices of their products to maximize their profits; that is, for $i=\{1,2\}$,

$$
\begin{equation*}
\max _{w_{i}^{o}} \pi_{i}^{o}=w_{i}^{o} D_{i}^{o} \tag{9}
\end{equation*}
$$

Substituting the optimal wholesale prices into the retail prices, we obtain the optimal retail prices, and then the equilibrium demand for each product and the profits for the platform and the sellers.

Comparing the sellers' profits between the benchmark and the own brand scenarios, we summarize the impact of the platform's own brand introduction on the incumbent sellers under the sell-to contract in the following proposition.

Proposition 2. Under the sell-to contract, introduction of the platform's own brand doesn't always hurt the incumbent sellers. The incumbent sellers can be better off, i.e., $\pi_{i}^{o}>\pi_{i}^{s}, i \in\{1,2\}$, if and only if $a<a_{2}$ and $\delta<\delta_{2}$.

Proposition 2 is partially in line with the intuition of Part (i) of Proposition 1: the incumbents cannot benefit from the own-brand introduction when this new entrant is too competitive or a significant threat. Proposition 2 also highlights the importance of pricing contracts for the incumbent sellers in the competition against the platform's own brand. Double marginalization hurts the incumbent sellers under the sell-to contract even in benchmark scenario $s$ where there is no platform's own brand product. With the platform's own brand introduction, the price competition becomes more intense. Ceteris paribus, the platform can market its product with a lower price than incumbents' products since the retail prices for the sellers' products are set with a positive markup on their wholesale prices. As $a$ increases, the platform benefits more significantly by increasing its profit margins. The platform's marketing advantage resulting from the dual role of the participating in and operating the marketplace can give its own brand unfair competitive advantage, exacerbating the incumbent sellers' already dire situation under the sell-to contract.

Perhaps, the established results that the traditional reseller's introduction of its own brand product is harmful to the incumbent sellers prompt the politicians and policy makers alike to call for anti-trust law regulation prohibiting Amazon (an online platform) from introducing its own brand product. It would be of great interest to the policy makers to ascertain whether such regulation is effective in protecting the incumbent sellers, which undoubtedly depends on the e-commerce platform's strategic response to the proposed ban of its own brand product. Facing the ban, one possible move of the e-commerce platform to increase its profit is to introduce a new brand. We analyze the impact of this strategic move on the incumbent sellers in the next subsection. We then examine the e-commerce platform's overall strategic choice in terms of its preferences for the own brand launch and pricing contracts in Section 5.

### 4.3 Platform Introducing a New Brand in Place of its Own Brand

One common strategic response to regulation banning the platform's own brand is to introduce a new reputable brand as a surrogate of the platform's own brand so as to increase its profit. Hence, we consider a new brand $n$ selling its product $n$ in the platform's marketplace. We use a superscript $n$ to denote scenario $n$, and a subscript $n$ to denote this new brand. For a meaningful comparative analysis, we assume the same characteristics of the platform's own brand for this new brand. That is, the base level demands for the new brand $n$ and the incumbent sellers' products $i$ are $a(a \geq 0)$ and 1 , respectively, and $\delta$ is the cross price sensitivity between products $n$ and $i, i \in\{1,2\}$. Therefore, $D_{n}^{n}$ and $D_{i}^{n}$ of scenario $n$ are analogous to $D_{r}^{o}$ and $D_{i}^{o}$ of scenario $o$ in Equation (2), respectively.

We now derive the subgame perfect equilibria in scenario $n$ under the sell-on contract. In stage 1 of the game, three sellers (two incumbents plus one new brand) determine optimal retail prices to maximize their profits given the commission $\alpha$ :

$$
\begin{align*}
& \max _{p_{i}^{n}} \pi_{i}^{n}=(1-\alpha) p_{i}^{n} D_{i}^{n}  \tag{10}\\
& \max _{p_{n}^{n}} \pi_{n}^{n}=(1-\alpha) p_{n}^{n} D_{n}^{n} \tag{11}
\end{align*}
$$

where $i \in\{1,2\}$. The profit of the e-commerce platform is $\pi_{r}^{n}=\alpha\left(p_{1}^{n} D_{1}^{n}+p_{2}^{n} D_{2}^{n}+p_{n}^{n} D_{n}^{n}\right)$.

Following similar analyses as in subsection 4.1, we derive the equilibrium outcome. In the following proposition, we discuss the effect of having another new brand $n$, in place of the platform's own brand, on the two incumbent sellers under the sell-on contract.

Proposition 3. Under the sell-on contract,
(a) introduction of another new brand doesn't always hurt the incumbent sellers, i.e., $\pi_{i}^{n}>\pi_{i}^{s}$, $i \in\{1,2\}$, if and only if (i) $a<a_{31}$ and $\delta<\delta_{3}$ or (ii) $a>a_{32}$;
(b) the incumbent sellers always profit more when they compete with the platform's own brand than with another new brand, i.e., $\pi_{i}^{o}>\pi_{i}^{n}, i \in\{1,2\}$.

The intuition behind part (a) of Proposition 3 is analogous to that in Proposition 1. The encroachment of product $n$ into the incumbents' market is more significant when the base level demands of the products are more comparable (i.e., $a$ is closer to 1 ). This effect is common across the scenarios $n$ and $o$. However, different from Proposition 1, when the incumbents compete with another brand $n$, the cooperative nature between the platform and the two incumbents no longer exists. That is, in scenario $o$, rather than in scenario $n$, the incumbents can be better off under the sell-on contract, a key finding presented in part (b) of Proposition 3.

It is worth noting that the new brand introduction can be more profitable for the incumbent sellers (i.e., $\pi_{i}^{n}>\pi_{i}^{s}$ ) even though it adds another competitor. This is true when this new brand can be away from their head-on competition as stated in part (a) of Proposition 3, the similar intuition that applies to the scenario of the platform's own brand introduction as described in Proposition 1. However, we find that the incumbent sellers are always better off in the own brand scenario than the new brand scenario (i.e., $\pi_{i}^{o}>\pi_{i}^{n}$ ). It is because in contrast to scenario $n$ where only the competitive nature prevails, the cooperative nature also plays a role in scenario $o$ as a portion of the platform's profit comes from the incumbent sellers.

Next, we derive the subgame perfect equilibria in scenario $n$ under the sell-to contract. In stage 2 of the game, the platform determines optimal the retail price for each of the three products simultaneously to maximize its profit:

$$
\begin{equation*}
\max _{p_{1}^{n}, p_{2}^{n}, p_{n}^{n}} \pi_{r}^{n}=\left(p_{1}^{n}-w_{1}^{n}\right) D_{1}^{n}+\left(p_{2}^{n}-w_{2}^{n}\right) D_{2}^{n}+\left(p_{n}^{n}-w_{n}^{n}\right) D_{n}^{n} . \tag{12}
\end{equation*}
$$

In the stage 1, anticipating the retail prices, the three sellers simultaneously determine optimal wholesale prices to maximize their own profits:

$$
\begin{align*}
& \max _{w_{i}^{n}} \pi_{i}^{n}=w_{i}^{n} D_{i}^{n}  \tag{13}\\
& \max _{w_{n}^{n}} \pi_{n}^{n}=w_{n}^{n} D_{n}^{n} \tag{14}
\end{align*}
$$

where $i \in\{1,2\}$. The following proposition describes the effect of another new brand $n$, in place of the platform's own brand, on the two incumbent sellers under the sell-to contract.

Proposition 4. Under the sell-to contract,
(a) introduction of another new brand doesn't always hurt the incumbent sellers, i.e., $\pi_{i}^{n}>\pi_{i}^{s}$, $i \in\{1,2\}$, if and only if (i) $a<a_{31}$ and $\delta<\delta_{3}$ or (ii) $a>a_{32}$;
(b) the incumbent sellers always profit less when they compete with the platform's own brand than with another new brand, i.e., $\pi_{i}^{o}<\pi_{i}^{n}, i \in\{1,2\}$.

The intuition of part (a) in Proposition 4 is analogous to that of Proposition 3(a) because the essence of added competition is not different across contracts in scenario $n$. However, under the sell-to contract, as presented in part (b) of Proposition 4, who competes with the incumbent sellers makes a critical difference across scenarios. While the new brand $n$ plays a simple role as a competitor in scenario $n$, the own brand platform in scenario $o$ not only strives to squeeze out more margins from the incumbent sellers but also takes an aggressive pricing strategy to defeat the sellers in product competition under the sell-to contract. Taking Propositions 3(b) and 4(b) together, we show in the following corollary that it is the best for the sellers to compete with the platform's own brand under the sell-on contract but the worst for them to do so under the sell-to contract.

Corollary 1. The incumbent sellers always profit more (less) when they compete with the platform's own brand than with another new brand under the sell-on (sell-to) contract; moreover, their profits are higher under the sell-on than sell-to contract (i.e., $\pi_{i}^{o, S O}>\pi_{i}^{n, S O}>\pi_{i}^{n, S T}>\pi_{i}^{o, S T}, i \in\{1,2\}$ ).

Note that we use another superscripts of $S O$ and $S T$ for the sell-on and sell-to contracts respectively. Corollary 1 indicates that, in contrast to the sell-on contract, the own brand scenario
under the sell-to contract is the worst for the incumbent sellers under most market conditions (i.e., $\left.\min \left\{\pi_{i}^{n, S T}, \pi_{i}^{s, S T}\right\}>\pi_{i}^{o, S T}\right)$. Our finding offers an important implication to the policy makers aiming to protect the incumbent sellers. Prohibiting the platform from introducing its own brand is more urgent not under the sell-on contract as in Senator Warren's call to ban the dual roles of Amazon, but rather under the sell-to contract. Interestingly, scenario $n$ works the best to protect the incumbent sellers even if the new-brand $n$ highly outperforms or underperforms the incumbents; otherwise, under the sell-to contract, the best option to protect the incumbents is to allow no new additional competitors in the marketplace (i.e., scenario $s$ ), an option that may be too extreme and anti-competitive.

We examine the impact of the platform introducing its own brand or a new brand (in the event that the own brand introduction is banned) on the incumbent sellers. Surprisingly, the incumbent sellers are not always hurt by the seemingly harmful introduction of the own brand or a new brand, and the effect on the incumbents critically depends on the pricing contracts. The analyses and results thus far take the pricing contract as given and assume that the platform has introduced a new brand or its own brand as in the case of Amazon. In the next section, we study the platform's incentive and preference for introducing its own brand or a new brand, and the type of contracts.

## 5 Platform's Incentive and Preference for Own Brand and Contract Alternatives

In subsection 5.1, we first establish whether the platform has an incentive to introduce its own brand, a new brand, or do without either under different types of contracts by comparing its profit under each scenario. We find that the platform has a strong incentive to introduce its own brand since it generates the higher profit under both the sell-on and sell-to contracts. We next derive the conditions under which the platform prefers the sell-on contract with the introduction of its own brand, and show that the platform and the incumbent sellers unfortunately have divergent regions of preferences for the sell-on contract. That is, if the platform is allowed to dictate which type of contract (sell-on or sell-to) to adopt for the incumbent sellers, the sellers are always worse off with
the own brand introduction when the platform prefers the sell-on contract.
We next identify the platform's strategic response in subsection 5.2 in the event of three possible regulations- barring the platform from introducing its own brand only under the sell-on contract as in Senator Warren's proposal, only under the sell-to contract, or under both the sell-on and sell-to contracts. Subsection 5.2 offers important and useful insights for the policy makers regarding the effectiveness of these policies aimed at protecting the incumbent sellers in the platform's marketplace.

### 5.1 Platform's Incentive for Own Brand Introduction

The platform has three options of branding decision for the sell-on contract and the sell-to contract: to introduce its own brand (scenario o), to introduce a new brand (scenario $n$ ), or do without either (the benchmark scenario s). In the event of policy makers prohibiting the platform from own brand introduction, the platform is left with only scenario $n$ and the benchmark scenario $s$, and the impact of scenario $n$ on the incumbent sellers has been analyzed in subsection 4.3. To compare the profits of all three scenarios, we maintain the same superscripts for the scenarios and use another superscripts of $S O$ and $S T$ for the sell-on and sell-to contracts respectively. The platform's preference for brand choice under each contract type is summarized in the following proposition.

Proposition 5. (a) Under both contracts, the platform has an incentive to introduce its own brand as long as the base level demand of its brand is not too low, i.e., $\pi_{r}^{o, S O}>\pi_{r}^{s, S O}$ if and only if $a>a_{51}$ under the sell-on contract and $\pi_{r}^{o, S T}>\pi_{r}^{s, S T}$ if and only if $a>a_{52}$ under the sell-to contract;
(b) it is always more profitable with its own brand than another brand; that is, $\pi_{r}^{o, S O}>\pi_{r}^{n, S O}$ under the sell-on contract and $\pi_{r}^{o, S T}>\pi_{r}^{n, S T}$ under the sell-to contract.

Proposition 5 clearly shows the platform's strong incentives to introduce its own brand product and to exercise its marketing maneuver in favor of its own brand. Note that the thresholds of $a$ for the platform to profit more in scenario $o$ are always less than 1 (i.e., $a_{51}<1$ and $a_{52}<1$ ), which is the base level demand of the incumbent sellers. This finding is consistent with the recent trend of Amazon's aggressive expansion of its own brand business and unfair privilege. A natural question then arises: Which contract type generates a higher profit for the platform with the introduction
of its own brand? The incumbent sellers are obviously concerned with the platform's preference for the contract type in the presence of the own brand product since there can be a win-win situation as shown in Propositions 1 and 2 and the own brand product affects their profits differently across pricing contracts. The following proposition describes the conditions for the platform's preference for the sell-on contract when its own brand is introduced.

Proposition 6. When the platform chooses the scenario o, where its own brand introduction is the most profitable, i.e., $\pi_{r}^{o, S O}>\max \left\{\pi_{r}^{s, S O}, \pi_{r}^{n, S O}\right\}$ and $\pi_{r}^{o, S T}>\max \left\{\pi_{r}^{s, S T}, \pi_{r}^{n, S T}\right\}$,
(a) the platform prefers the sell-on contract (i.e., $\pi_{r}^{o, S O}>\pi_{r}^{o, S T}$ ) if and only if $a<a_{6}$ and $\alpha>$ $\alpha_{6}$; however, the incumbent sellers always prefer the sell-on contract (i.e., $\pi_{i}^{o, S O}>\pi_{i}^{o, S T}, i \in\{1,2\}$ );
(b) no matter which contract the platform prefers, its own brand introduction always hurts the incumbent sellers (i.e., $\pi_{i}^{o, S O}<\pi_{i}^{s, S O}$ and $\pi_{i}^{o, S T}<\pi_{i}^{s, S T}$, $i \in\{1,2\}$ ).

Proposition 6(a) shows that the own brand platform prefers the sell-on contract to the sell-to contract when its portion of the revenue from the sellers is significant (as indicated by a high $\alpha$ ) and the platform's own brand does not have a high base level demand represented by a low $a$. In other words, when the relative advantage of the platform's own brand product is weak, the platform prefers the sell-on contract if it can increase the portion of its profit from the sellers.

Overall, the sell-to contract is more likely to be attractive to the platform. Other things being equal, the platform can take a low-price strategy compared to the incumbents and as the difference in base level demands increases (i.e., high $a$ ), the platform can increase its markup significantly from not only its product but also those of the incumbent sellers at the expense of the sellers. This tendency is more prominent only under the sell-to contract mainly because under the sellon contract, the platform's profitability is not completely opposite to the incumbents' due to the cooperative nature of the sell-on contract. Under the sell-on contract, the platform needs to consider both sources of profits, one from the profit of its own brand and the other from the incumbent sellers. When the relative advantage of the platform' own brand is strong (i.e., high $a$ ), the platform always benefits from its own brand introduction (i.e., Proposition 5(a)); however, the positive effect tends to be higher under the sell-to contract, as shown in part (a) of Proposition 6. Moreover, in scenario $o$, the contract preference of the incumbent sellers does not always align with that of the platform.

While the platform prefers the sell-on contract only when base level demand of the own brand is not high and the commission rate is high, the incumbent sellers always prefer the sell-on contract.

We find that with the platform's own brand introduction, the win-win condition for the incumbent sellers to be better off under the sell-on contract identified in Proposition $1\left(a>a_{12}\right)$ is unfortunately incompatible with that for the platform to choose the sell-on contract in part (a) of Proposition 6 (i.e., $a<a_{6}$ ) since $a_{6}<a_{12}$ and $a_{11}<a_{51}$ (see A5 and A6 of Online Appendix for details). Likewise, under the sell-to contract, we also find the incompatible conditions (i.e., $a_{2}<a_{6}$ ). Part (b) of Proposition 6 further indicates that the incumbent sellers are always worse off in the region of conditions that lead to the platform to opt for the sell-on or sell-to contract.

The key results of six propositions thus far paint the following overall picture. Propositions 1 and 2 specify the market conditions for each pricing contract under which the incumbent sellers can be better off, given the presence of the platform's own brand. Propositions 3 and 4 show that the same pattern holds for the case where the e-commerce platform introduces a new brand exhibiting the same characteristics of its own brand. Moreover, Corollary 1 summarizes that the incumbent sellers profit more from competing with the platform's own brand than with the new brand under the sell-on contract, while earning less profit under the sell-to contract. Finally, Propositions 5 indicates that the e-commerce platform has a strong incentive to introduce its own brand product for both the sell-on and the sell-to contracts. More importantly, Proposition 6 highlights the incongruence of conditions between the e-commerce platform's preference for the sell-on contract and that of the incumbent sellers in scenario $o$. It further shows that with the introduction of the platform's own brand, the e-commerce platform will choose the sell-on contract under the conditions that only make the sellers worse off. We note that the results from Propositions 1 and 3-6 are new and significant to both the academia and the industry in understanding the impact of the e-commerce platform's introducing own brand on the incumbent sellers.

Propositions $1-6$ taken together underscore the importance of taking into account the ecommerce platform's incentive and preference on its brand positioning ( $a$ ), the competition in the category $(\delta)$, and the contracts with the incumbents, when the policy makers examine the impact of platform's introducing its own brand product. Without doing so, incumbent sellers on the surface
can be better off under the sell-on contract commonly adopted in the e-commerce marketplace according to Propositions 1 and 3, but Proposition 6 shows that the e-commerce platform will only choose the sell-on contract under conditions to the detriment of the incumbent sellers. Hence, the policy makers must consider the e-commerce platform's strategic response to whatever policy they develop with an aim to protect the incumbent sellers. The next subsection offers helpful insights into the effectiveness of three possible regulations that policy makers can implement to protect the incumbent sellers from the harm of the e-commerce platform's own brand introduction.

### 5.2 Platform's Response to Ban of Selling Own Brand in its Marketplace

There are essentially three possible regulations of prohibiting the e-commerce platform from introducing its product given two types of pricing models - prohibition of the platform's own brand only under the sell-on contract (identical to that proposed by Senator Warren), only under the sell-to contract, and under both types of contracts. We analyze the platform's strategic response to each of the three regulations and assess the effectiveness of each policy in this subsection. We examine the first policy option in the next proposition.

Proposition 7. The platform's best response to the ban of introducing its own brand only under the sell-on contract is as follows:
(a) keeping only the sell-on contract with the two incumbents (i.e., $\pi_{r}^{s, S O}>\pi_{r}^{o, S T}$ and $\pi_{r}^{s, S O}>\pi_{r}^{n, S O}$ ) if and only if $a<a_{7}$ and $\alpha>\alpha_{7}$. In this case, the policy helps the incumbent sellers earn more profit than they do without such a policy (i.e., $\pi_{i}^{s, S O}>\pi_{i}^{o, S O}, i \in\{1,2\}$ ).

Otherwise,
(b) keeping only the sell-to contract and introducing its own brand (i.e., $\pi_{r}^{o, S T}>\pi_{r}^{s, S O}$ and $\left.\pi_{r}^{o, S T}>\pi_{r}^{n, S O}\right)$. In this case, the policy does not help the incumbent sellers earn more profit than they do without such a policy (i.e., the sellers' profit is unaffected if the platform prefers the sell-to contract when there is no policy; however, the policy hurts the incumbent sellers if the platform prefers the sell-on contract when there is no policy as $\left.\pi_{i}^{o, S T}<\pi_{i}^{o, S O}, i \in\{1,2\}\right)$.

The intuition behind Proposition 7(a) is partly analogous to that of Propositions 5 and 6 . When the base level demand of the platform's own brand product or the new brand product is
small, the platform prefers not to introduce the additional product. Moreover, once the platform can increase its profit from the sellers through a higher commission rate, it would prefer the sell-on contract. In this case, the platform would introduce its own brand and keep the sell-on contract (i.e., $\pi_{r}^{o, S O}>\pi_{r}^{o, S T}$ ) if there is no ban of introducing own brand under the sell-on contract. In this light, the policy helps the incumbent sellers earn more than they do without such a policy (i.e., $\left.\pi_{i}^{s, S O}>\pi_{i}^{o, S O}, i \in\{1,2\}\right)$.

More importantly, Proposition 7(b) indicates that as long as the own brand is not very weak, the platform's profit of introducing its own brand under the sell-to contract is greater than that in both the scenario where there is no own brand under the sell-on contract and the scenario where it introduces a new brand under the sell-on contract. In other words, the platform's best response to the type of regulation proposed by Senator Warren is very likely to adopt the sell-to contract for the incumbent sellers and to introduce its own brand. In this case, the platform would have chosen to introduce its own brand under the sell-on or sell-to contract if there is no policy. The sellers' profit is unaffected by the policy if the platform would choose to introduce its own brand under the sell-to contract even when there is no policy. However, the policy hurts the sellers if the platform would choose to introduce its own brand under the sell-on contract if there is no policy (i.e., $\pi_{i}^{o, S T}<\pi_{i}^{o, S O}, i \in\{1,2\}$ ). Therefore, the policy of prohibiting the e-commerce platform from introducing its own brand only under the sell-on contract does not accomplish the goal of protecting the incumbent sellers and potentially leads to an outcome that hurts them instead. We next present the result when the ban is implemented only under the sell-to contract.

Proposition 8. The platform's best response to the ban of introducing its own brand only under the sell-to contract is as follows:
(a) keeping only the sell-to contract with the two incumbents (i.e., $\pi_{r}^{s, S T}>\pi_{r}^{o, S O}$ and $\pi_{r}^{s, S T}>\pi_{r}^{n, S T}$ ) if and only if $a<\min \left(a_{81}, a_{82}\right), \theta>\theta_{8}$, and $\alpha<\alpha_{8}$. In this case, the policy helps the incumbent sellers earn more profit than they do without such a policy (i.e., $\pi_{i}^{s, S T}>\pi_{i}^{o, S T}, i \in\{1,2\}$ ).

Otherwise,
(b) keeping only the sell-on contract and introducing its own brand (i.e., $\pi_{r}^{o, S O}>\pi_{r}^{s, S T}$ and $\left.\pi_{r}^{o, S O}>\pi_{r}^{n, S T}\right)$. In this case, the policy helps the incumbent sellers earn more profit than they do
without such a policy (i.e., the sellers' profit is unaffected if the platform prefers the sell-on contract when there is no policy; also, the policy helps the incumbent sellers earn more profit if the platform prefers the sell-to contract when there is no policy as $\left.\pi_{i}^{o, S O}>\pi_{i}^{o, S T}, i \in\{1,2\}\right)$.

The intuition behind Proposition 8(a) further points out the importance of recognizing the nature of competition and the effect of own brand, both of which work differently in the two pricing contracts. When the incumbent sellers are more competitive with each other indicated by a high $\theta$, the platform can squeeze more profit out of them, which is feasible only under the sell-to contract. Under the sell-on contract, however, the competition between incumbent sellers makes the profit from them less critical and the sell-on contract less attractive to the platform. Specifically, under the sell-on contract, the own brand platform can be better off when the incumbents' competition is not significant (i.e., low $\theta$ ) because the marginal benefit from introducing its own brand (i.e., additional competition), compared to the sellers-only market, is higher when their competition is lower. If the competition between the incumbent sellers is fierce (i.e., high $\theta$ ), the negative effect of the platform's brand on price competition is more evident to the platform under the sell-on contract where all players compete on the retail prices. Furthermore, as the platform derives a portion of its profit from those of the sellers, the loss from the sellers' intense competition can lead to a significant reduction in the platform's profit under the sell-on contract. This becomes more serious as the platform's commission rate is low and its own brand only aggravates this already heated competition (i.e., low $\alpha$ and $a$, and high $\theta$ ). In this case, the platform prefers introducing its own brand and keeping the sell-to contract (i.e., $\pi_{r}^{o, S T}>\pi_{r}^{o, S O}$ ) if there is no ban of introducing own brand under the sell-to contract. Consequently, the policy helps the incumbent sellers because they earn more than they do without such a policy (i.e., $\pi_{i}^{s, S T}>\pi_{i}^{o, S T}, i \in\{1,2\}$ ).

Note that the platform with its own brand always benefits more from a higher $a$. Although the platform's gain can be higher under the sell-to contract as discussed in Proposition 6, the own brand ban only under the sell-to contract leads to the platform's switch of its preference to the sell-on contract, as indicated in part (b) of Proposition 8. In this case, the platform prefers to introduce its own brand under the sell-on or sell-to contract if there is no ban of introducing the own brand under the sell-to contract. The sellers' profit is unaffected by the policy if the platform prefers to
introduce its own brand under the sell-on contract even when there is no policy. Furthermore, the policy helps the sellers earn more profit if the platform prefers to introduce its own brand and adopt the sell-to contract (i.e., $\pi_{i}^{o, S T}<\pi_{i}^{o, S O}, i \in\{1,2\}$ ).

So far, we examine the platform's decisions in response to the policy banning its own brand introduction that applies to only one type of contracts. The next proposition summarizes the platform's strategic response to an all-out ban policy for both the sell-on and sell-to contracts.

Proposition 9. The platform's best response to the all-out ban of the own brand introduction under both contracts is
(a) keeping only the sell-on contract with the two incumbents (i.e., $\pi_{r}^{s, S O}>\pi_{r}^{s, S T}, \pi_{r}^{s, S O}>$ $\pi_{r}^{n, S T}$, and $\left.\pi_{r}^{s, S O}>\pi_{r}^{n, S O}\right)$ if and only if $a<\min \left(a_{91}, a_{92}\right)$ and $\alpha>\frac{\theta+1}{4}$. Or,
(b) keeping only the sell-to contract with the two incumbents (i.e., $\pi_{r}^{s, S T}>\pi_{r}^{s, S O}, \pi_{r}^{s, S T}>\pi_{r}^{n, S T}$, and $\left.\pi_{r}^{s, S T}>\pi_{r}^{n, S O}\right)$ if and only if $a<\min \left(a_{93}, a_{82}\right), \theta>\theta_{8}$, and $\alpha<\frac{\theta+1}{4}$.

In these two cases, the all-out ban helps the incumbent sellers earn more profit than they do without such a policy (i.e., If the platform prefers to introduce its own brand and adopt the sell-to contract when there is no policy, $\pi_{i}^{s, S O}>\pi_{i}^{o, S T}$ and $\pi_{i}^{s, S T}>\pi_{i}^{o, S T}$ hold; and if the platform prefers to introduce its own brand and adopt the sell-on contract when there is no policy, $\pi_{i}^{s, S T}>\pi_{i}^{o, S O}$ and $\pi_{i}^{s, S O}>\pi_{i}^{o, S O}$ hold, $\left.i \in\{1,2\}\right)$.

Otherwise,
(c) adding a new brand to compete with the two incumbents under the sell-on contract (i.e., $\pi_{r}^{n, S O}>\pi_{r}^{s, S T}, \pi_{r}^{n, S O}>\pi_{r}^{n, S T}$, and $\pi_{r}^{n, S O}>\pi_{r}^{s, S O}$ ) if and only if $\alpha>\alpha_{9}$. Or,
(d) adding a new brand to compete with the two incumbents under the sell-to contract (i.e., $\pi_{r}^{n, S T}>\pi_{r}^{s, S T}, \pi_{r}^{n, S T}>\pi_{r}^{n, S O}$, and $\pi_{r}^{n, S T}>\pi_{r}^{s, S O}$ ) if and only if $\alpha<\alpha_{9}$.

In these two cases, the all-out ban helps the incumbent sellers earn more profit only if the platform prefers to introduce its own brand under the sell-to contract when there is no policy (i.e., $\pi_{i}^{n, S O}>$ $\pi_{i}^{o, S T}$ and $\left.\pi_{i}^{n, S T}>\pi_{i}^{o, S T}, i \in\{1,2\}\right)$. However, the all-out ban hurts the incumbent sellers if the platform prefers to introduce its own brand under the sell-on contract when there is no policy (i.e., $\pi_{i}^{n, S O}<\pi_{i}^{o, S O}$ and $\left.\pi_{i}^{n, S T}<\pi_{i}^{o, S O}, i \in\{1,2\}\right)$.

Proposition 9(a) and 9(b) show that the platform tends to stay only with the two incumbents
without introducing another brand if the new brand does not have a high base level demand as indicated by the small threshold values of $a$. The intuition for the platform's incentive across contracts and market conditions is analogous to that of previous propositions. In those cases, the platform would have introduced its own brand if there was no all-out ban, and the policy helps the incumbent sellers by prohibiting the platform from introducing its own brand.

More importantly, Proposition 9(c) and 9(d) suggest that the draconian all-out ban covering all contracts can be effective in helping the incumbent sellers but the benefit to them is limited. The policy can even hurt the incumbent sellers if the platform prefers to introduce its own brand under the sell-on contract when there is no policy, in which the incumbent sellers can earn more profit than they do with any contract in scenario $n$ (i.e., $\pi_{i}^{n, S O}<\pi_{i}^{o, S O}$ and $\pi_{i}^{n, S T}<\pi_{i}^{o, S O}$, $i \in\{1,2\})$. Because of the same reason, although the all-out ban can help the incumbent sellers if the platform prefers to introduce its own brand under the sell-to contract when there is no policy (i.e., $\pi_{i}^{n, S O}>\pi_{i}^{o, S T}$ and $\pi_{i}^{n, S T}>\pi_{i}^{o, S T}, i \in\{1,2\}$ ), such a policy is less effective in protecting the incumbent sellers than the situation described in Proposition 8(b) (i.e., $\pi_{i}^{n, S O}<\pi_{i}^{o, S O}$ and $\left.\pi_{i}^{n, S T}<\pi_{i}^{o, S O}, i \in\{1,2\}\right)$.

In summary, the proposed policy of banning platform's own brand only under one contract can benefit or hurt the incumbent sellers. The harmful effect is more pronounced when the policy is only applied to the sell-on contract, which results in the platform adopting the worst scenario for the incumbent sellers (Proposition 7(b)). Such policy can help the incumbent sellers only when the own brand's relative potential is very small and the platform chooses to keep the sell-to contract with the two incumbents. Meanwhile, the ban under the sell-to contract can help the incumbent sellers in a better way since the sellers' preference can be better aligned with the platform's strategic move (Proposition 8(b)). Interestingly, this policy can help the sellers both when the base demand of platform's own brand is lower than the incumbent sellers' (i.e., keeping the sell-on contract with the two incumbents) and outperforms the incumbent sellers' (i.e., keeping the sell-on contract and introducing its own brand). While the all-out ban can also help the incumbent if the base demand of the platform's own brand is low, the benefit is limited and not as effective as in the case of the enforcement only under the sell-to contract if the base demand of the platform's own brand is high
(Proposition 9(c) and 9(d)).

## 6 Model Extensions

All of our analyses hitherto make some assumptions to reflect the reality and ensure fair and meaningful comparisons across all scenarios and contracts. We present a series of extensions in this section to show the robustness of our key findings. First, we assume that the commission rate is exogenous and the same across scenarios in our main analyses, to reflect the reality of fairly stable commission rates charged by the e-commerce platforms today. In subsection 6.1, we discuss the extension of the model to allow the commission rate to be a decision variable for the e-commerce platform. Second, in subsection 6.2, we discuss a new benchmark scenario where there are three incumbent sellers. Third, we assume that the base level demand of the own brand is exogenous in the main analyses. This assumption ensures that we make fair comparisons across different scenarios and contracts, emphasizing the key policy implications for two pricing contracts. In subsection 6.3, we discuss the platform's incentive on the base level demand of the own brand, and how different policy alternatives would affect its incentive and thus the profits of the incumbent sellers. Fourth, in subsection 6.4 , we discuss how the policy would help the sellers differently in two pricing contracts if the policy is to restrict the competition between the own brand and incumbent products. Fifth, subsection 6.5 presents the extension to consider the revenue sharing contract, a variation of the wholesale (i.e., sell-to) contract. Finally, we show that our main results and policy implications remain the same when demand functions are directly derived from consumer utility.

### 6.1 Platform's Incentive: Commission Rates

We extend our model to consider the platform's optimal decision on the commission rate, which becomes the decision before all other stages. We find that the monopolistic platform will increase the commission rate to the maximum possible value since $\frac{\partial \pi_{r}^{s}}{\partial \alpha}>0, \frac{\partial \pi_{r}^{o}}{\partial \alpha}>0$, and $\frac{\partial \pi_{r}^{n}}{\partial \alpha}>0$, and our main results carry over. For example, under the sell-on contract, the incumbent sellers can be better off from the platform's own brand introduction. In the extreme, if the platform can take all of the sales proceeds from the sellers $(\alpha=1)$, the sell-on contract will cease to exist because it amounts to no
sales for the sellers in the marketplace. If the commission rate is different across the scenarios (e.g., scenarios $o$ vs. $s$ ), our results are qualitatively the same if it is significantly lower in scenario $s$ than in scenario $o$. Overall, the intuition remains the same, considering the platform's incentives. From the policy perspective of protecting the incumbent sellers and satisfying the platform's incentive, the commission rate can be set to the value such that the sellers are indifferent between contracts and the platform prefers the sell-on to the sell-to contract. In this case, the results are qualitatively the same as well.

### 6.2 Another Benchmark with Three Incumbents

We now consider a new benchmark scenario, where there are three incumbent sellers. We compare sellers' profits in this new benchmark scenario (i.e., scenario ns) with those when the platform introduces its own brand (i.e., scenario $o$ ) or a new brand (i.e., scenario $n$ ), respectively. This group of analyses helps us further confirm that our conclusions are due to the difference between pricing contracts (i.e., sell-on vs. sell-to), not the difference (if any) between the two-sellers and three-sellers demand systems. Note that scenario $n s$ is a special case of scenario $n$ when $a=1$, and Appendices A3 and A4 describe the detailed derivations and equilibrium results of this new benchmark scenario with $a=1$. Without loss of generality, we assume that the platform can remove the third seller, and introduce its own brand (i.e., scenario $o$ ) or a new brand (i.e., scenario $n$ ). The results in this new benchmark scenario with sell-on contract are consistent with those in Propositions 1 and 3.

Proposition 10. In the new benchmark scenario with three incumbent sellers, under the sell-on contract,
(a) introduction of the platform's own brand doesn't always hurt the incumbent sellers; that is, $\pi_{i}^{o}>\pi_{i}^{n s}, i \in\{1,2\}$ if and only if (i) $a<a_{1}^{n s}$ or (ii) $a>a_{2}^{n s}$;
(b) introduction of another new brand doesn't always hurt the incumbent sellers; that is, $\pi_{i}^{n}>$ $\pi_{i}^{n s}, i \in\{1,2\}$ if and only if (i) $a<a_{3}^{n s}$ or (ii) $a>a_{4}^{n s}$.

Likewise, the results in this new benchmark scenario under the sell-to contract are also consistent with those in Propositions 2 and 4 as follows.

Proposition 11. In the new benchmark scenario with three incumbent sellers, under the sell-to contract,
(a) introduction of the platform's own brand doesn't always hurt the incumbent sellers; that is, $\pi_{i}^{o}>\pi_{i}^{n s}, i \in\{1,2\}$ if and only if $a<a_{5}^{n s}$;
(b) introduction of another new brand doesn't always hurt the incumbent sellers; that is, $\pi_{i}^{n}>$ $\pi_{i}^{n s}, i \in\{1,2\}$ if and only if (i) $a<a_{3}^{n s}$ or (ii) $a>a_{4}^{n s}$.

### 6.3 Platform's Incentive: Demand of its Own Brand

We extend our model to consider the platform's incentive on the base level demand of the own brand. Overall, we show that considering the platform's incentive and the policy perspective of protecting the incumbent sellers, the intuition of our analyses and policy implications remain the same as those in our main analyses. We first show that the platform always has the incentive to increase the base demand level of its own brand.

Proposition 12. (a) The platform always has an incentive to achieve a higher base level demand of its own brand when the base level demand is not too small. That is, $\frac{\partial \pi_{r}^{S O, o}}{\partial a}>0$ if and only if $a>a_{1}^{b l} ; \frac{\partial \pi_{r}^{S T, o}}{\partial a}>0$ if and only if $a>a_{2}^{b l}$.
(b) When there is no policy banning the own brand or when the policy bans it only under the sellon contract, as the platform always has an incentive to achieve a higher a, the incumbent sellers are always hurt from the increased base level demand of the own brand a (i.e., $\frac{\partial \pi_{i}^{S T, o}}{\partial a}<0, i \in\{1,2\}$ ). When the policy only bans it under the sell-to contract, the incumbent sellers can benefit from the increased base level demand of the own brand a (i.e., $\frac{\partial \pi_{i}^{S O, o}}{\partial a}>0, i \in\{1,2\}$ ) if and only if $a>\frac{4}{\delta}$.

Note that the thresholds value $a_{1}^{b l}$ and $a_{2}^{b l}$ are always less than 1. Part (a) of Proposition 12 is consistent with the recent trend of Amazon's aggressive expansion of its own brand business giving its own brand product more and more unfair competitive advantage (Thomson and Hansen, 2016; The Capital Forum, 2016). Proposition 12(b) is in line with the intuition of Propositions 1 and 2 in terms of the effects of the platform's own brand product on the incumbent sellers, further emphasizing the different policy implications in different pricing contracts.

We further discuss the platform's decision on $a$ and the related policy implications by considering a situation in which the platform needs to set $a$ such that the incumbent sellers would not deviate to a different pricing contract. Since $\pi_{i}^{o, S O}<\pi_{i}^{o, S T}$ if and only if $a<\underline{a}^{\bullet}$ and $\alpha>\underline{\alpha}^{\bullet}$ as discussed in the Proof of Proposition 9 in Appendix A9, the platform would set $a$ as $\underline{a}^{\bullet}$ under the sell-to contract and set $a$ as high as possible under the sell-on contract. Once the policy bans the own brand only under the sell-on contract, the platform would set $a$ as high as possible under the sell-to contract because the incumbent sellers do not have other contract to choose in scenario o and they would always suffer (i.e., $\frac{\partial \pi_{i}^{S T, o}}{\partial a}<0, i \in\{1,2\}$ ). While the platform has the incentive to set $a$ as high as possible under the sell-on contract when there is no policy or the policy only bans the own brand under the sell-to contract, our results indicate that (i) increasing $a$ hurts the incumbent sellers less under the sell-on contract than under the sell-to contract (i.e., $\frac{\partial \pi_{i}^{S O, o}}{\partial a}>\frac{\partial \pi_{i}^{S T, o}}{\partial a}, i \in\{1,2\}$ ) when $a$ is not too small; and (ii) the incumbent sellers can even benefit from the higher base level demand of the own brand (i.e., $\frac{\partial \pi_{i}^{S O, o}}{\partial a}>0, i \in\{1,2\}$ ) when $a$ is high enough (i.e., $a>\frac{4}{\delta}$ ).

### 6.4 Competition Between the Platform and Incumbents

We extend our analyses to examine the impact of the policy limiting the competition intensity between the own brand and the incumbent sellers. For example, the policy can limit the platform from using the data of the incumbent sellers for the anti-competitive purpose, in terms of how to shape and promote its own brand product (e.g., CNBC, 2019). The policy can prohibit the platform from directly promoting its own brand right next to the competing incumbent sellers' products or restrict the platform from having a similar product as the incumbent sellers' products. In general, these policies tend to protect the incumbent sellers by decreasing the degree of cross price sensitivity, and we show that such policies also work differently in two pricing contracts.

Proposition 13. The policy of limiting the competition between the own brand and the incumbents affects the incumbent sellers differently across the sell-on and sell-to contracts. That is,
(a) Under the sell-on contract, the incumbent sellers can be hurt as the degree of the cross price sensitivity $\delta$ decreases (i.e., $\frac{\partial \pi_{i}^{S O, o}}{\partial \delta}>0, i \in\{1,2\}$ ) if and only if $a>a^{c p}$.
(b) Under the sell-to contract, the incumbent sellers always benefit as the degree of the cross
price sensitivity $\delta$ decreases (i.e., $\frac{\partial \pi_{i}^{S T, o}}{\partial \delta}<0, i \in\{1,2\}$ ).

### 6.5 Impact under Revenue Sharing

We now consider a variation of the sell-to contract, the revenue sharing contract. Similar to the sellto contract, in stage 1 of the game, the incumbent sellers set the wholesale prices and the platform decides the optimal retail prices in stage 2 of the game. For instance, in the benchmark scenario $s$, in stage 2 of the game, the platform's decision of optimal retail prices for each product is:

$$
\begin{equation*}
\max _{p_{1}^{s}, p_{2}^{s}} \pi_{r}^{s}=\left(p_{1}^{s} \gamma-w_{1}^{s}\right) D_{1}^{s}+\left(p_{2}^{s} \gamma-w_{2}^{s}\right) D_{2}^{s} \tag{15}
\end{equation*}
$$

where $D_{i}^{s}, i \in\{1,2\}$, is defined in Equation (1) and $\gamma$ is the revenue sharing portion that belongs to the platform. Solving the first-order conditions, we derive the retail price of product $i$ as a function of the wholesale price of product $i$. In stage 1 of the game, anticipating the platform's reaction functions, the two sellers derive their optimal wholesale prices:

$$
\begin{equation*}
\max _{w_{i}^{s}} \pi_{i}^{s}=w_{i}^{s} D_{i}^{s}+(1-\gamma) p_{i}^{s} D_{i}^{s} \tag{16}
\end{equation*}
$$

Solving the first-order conditions simultaneously, we derive the equilibrium wholesale prices, retail prices and profits. The following proposition summarizes the effect of the platform's own brand introduction on the two incumbent sellers under the revenue sharing contract. The results under revenue sharing are consistent with those under the sell-to contract as in Propositions 2 and 4.

Proposition 14. Under the revenue sharing contract,
(a) introduction of the platform's own brand doesn't always hurt the incumbent sellers; that is, $\pi_{i}^{o}>\pi_{i}^{s}, i \in\{1,2\}$ if and only if $a<a_{1}^{r s}$ and $\delta<\delta_{1}^{r s}$;
(b) introduction of another new brand doesn't always hurt the incumbent sellers; that is, $\pi_{i}^{n}>\pi_{i}^{s}$, $i \in\{1,2\}$ if and only if (i) $a<a_{2}^{r s}$ and $\delta<\delta_{2}^{r s}$ or (ii) $a>a_{3}^{r s}$;
(c) the incumbent sellers always profit less when they compete with the platform's own brand than with another new brand; that is, $\pi_{i}^{o}<\pi_{i}^{n}, i \in\{1,2\}$.

Overall, double marginalization drives the results in the revenue sharing contract as in the wholesale contract, although its negative effect is alleviated to some extent by the revenue sharing.

### 6.6 Spokes Model: Consumer Utility Driven Demand Modeling

In this paper, we consider the impact of the platform introducing an own brand product from the aggregate market demands perspective. In this subsection, we extend our analysis to the utilitydriven models to show that our results are established based upon a consistent framework. We verify that our key results continue to hold in the utility-driven model. ${ }^{3}$

We consider the spokes model, a well-established framework that extends the classical Hotelling duopoly model to allow for multiple product sellers competing with all others (Chen and Riordan, 2007). More specifically, starting at the midpoint (centre) of a line of unit length, add lines of one-half length to form a radial network of $\mathrm{K}(\geq 2)$ lines (spokes). Each spoke, denoted as $l_{k}$, terminates at the centre and originates at the other end. Each spoke is of length $\frac{1}{2}$. The product $k$, or the seller $k$, is located at the origin of spoke $k$.

Consumers are located uniformly on the network of spokes and incur unit cost $t$ as in the standard Hotelling model. We assume that the number of consumers on each spoke is $\frac{1}{3}$ in both scenarios $s$ and $o$; that is, a benchmark scenario $s$ with two incumbent sellers has three spokes 1 , 2 , and $e$ (empty), and the market with 3 sellers with the own-brand entry in scenario $o$ has three spokes 1,2 , and $r$ (the own-brand). For a consumer on spoke $k$, seller $k$ is the first preference and others are equally likely her second preference. For instance, in scenario $s$, a consumer, with her second preference of the spoke 2 , located on spoke 1 at the distance $x$ from product 1 , is indifferent between purchasing product 1 and product 2 if $v_{1}-t x-p_{1}=v_{2}-t(1-x)-p_{2}$, where $v_{k}$ is the valuation obtained from product $k, p_{k}$ is retail price of product $k$, and $t$ is the unit cost, $k \in\{1,2, e\}$, whereas a consumer located on spoke 1 , with her second preference of the spoke $e$, buys product 1 as the spoke $e$ is empty. The consumers located in the spoke $e$ are absorbed equally to the demand for products 1 and 2. Thus, demand functions of products 1 and $2, D_{c k}^{s}, k \in\{1,2, e\}$, are

[^2]\[

$$
\begin{align*}
& D_{c 1}^{s}=\frac{1}{2}+\frac{v_{1}-v_{2}-p_{1}+p_{2}}{6 t}  \tag{17}\\
& D_{c 2}^{s}=\frac{1}{2}+\frac{v_{2}-v_{1}-p_{2}+p_{1}}{6 t}
\end{align*}
$$
\]

Likewise, in scenario $o$, she is also indifferent from purchasing product 1 and purchasing product $r$ if $v_{1}-t x-p_{1}=v_{r}-t(1-x)-p_{r}$. Demand functions of products 1,2 , and $r, D_{c k}^{o}, k \in\{1,2, r\}$, are

$$
\begin{align*}
& D_{c r}^{o}=\frac{1}{3}+\frac{2 v_{r}-v_{1}-v_{2}-2 p_{r}+p_{1}+p_{2}}{6 t} \\
& D_{c 1}^{o}=\frac{1}{3}+\frac{2 v_{1}-v_{r}-v_{2}-2 p_{1}+p_{r}+p_{2}}{6 t}  \tag{18}\\
& D_{c 2}^{o}=\frac{1}{3}+\frac{2 v_{2}-v_{r}-v_{1}-2 p_{2}+p_{r}+p_{1}}{6 t}
\end{align*}
$$

The demand functions in scenario $n$ are analogous to those in scenario $o$. For exposition ease and tractability, in each scenario we assume the existence of no-comparison consumers who buys from one firm as long as the utility is nonnegative (e.g., Iyer et al., 2005; Kwark et al., 2014). Therefore, demand of product $k$ is denoted as $D_{k}^{s}=D_{c k}^{s}+D_{l k}^{s}$ in scenario $s$, and as $D_{k}^{o}=D_{c k}^{o}+D_{l k}^{o}$ and $D_{k}^{n}=D_{c k}^{n}+D_{l k}^{n}$ in scenarios $o$ and $n$ respectively. ${ }^{4}$ As in our baseline model, we assume that the valuations of the two incumbents are the same and use the same notation for the own brand product (i.e., $v_{1}=v_{2}=1$ and $v_{r}=a$ ). All other model parameters remain the same as in our baseline model. For example, replacing demand functions in Equations (1) and (2) with $D_{k}^{s}$ and $D_{k}^{o}$ based on Equations (17) and (18), we derive the equilibrium profits for scenarios $s$ and $o$, respectively, in both sell-on and sell-to contracts. All equilibrium outcomes are delegated to Online Appendix A16.

The results show that the key messages and insights in our baseline model carry over to the case using utility-driven demand functions (Chen and Riordan, 2007). First, consistent with Propositions 1 and 2, the introduction of the platform's own brand doesn't always hurt the incumbent sellers under both contracts; however, the incumbent sellers always profit more when they compete with the platform's own brand than with another brand only under the sell-on contract, which is consistent with Propositions 3 and 4. In addition, consistent with Corollary 1, the incumbent sellers always profit more (less) when they compete with the platform's own brand than with another new brand

[^3]under the sell-on (sell-to) contract, and their profits are higher under the sell-on than sell-to contract. Second, consistent with Propositions 5 and 6, the platform has an incentive to launch its own brand product when its valuation $a$ is relatively high in both contracts; and no matter which contract the platform prefers, its brand entry decisions hurt the incumbent sellers. Third, in terms of the policy effectiveness and the platform's responses to the ban of introducing its own brand under the sell-on contract, the ban of introducing the own brand under the sell-to contract, and the all-out ban, we find consistent results as described in Propositions 7, 8, and 9. Specifically, the ban on the platform's brand entry under the sell-on contract does not necessarily help the incumbent sellers, but the ban under the sell-to contract can help the sellers earn more profits. While the all-out ban can also help the incumbent sellers, it is not as much as in the case of the ban only under the sell-to contract.

## 7 Conclusion

Sales on the e-commerce platform in the United States have experienced explosive growth over the years and are projected to surpass 740 billion in 2023 (Statista, 2019). There are two unmistakable trends of the e-commerce platform: (1) the expansion of its traditional role as a reseller into an online marketplace, where merchants directly sell products under their own control of pricing and pay a commission fee to the platform, and (2) the platform's introduction of its own brand product. As an example of the first trend, sellers in the U.S. Amazon marketplace offer over 350 million products, accounting for sixty five percent of the Amazon's total sales (Internet Retailer Research, 2017). A prominent example of the second trend is that Amazon's own brand product sales are expected to grow at a fast pace over the next five years and post $\$ 31$ billion in sales by 2022, according to Robinson Humphrey of SunTrust (Business Insider, 2017a). Amazon's doubling down on its own brand business has stoked a huge fear among the incumbent sellers on the marketplace (CNBC, 2018; Bloomberg, 2019). The reported unfair anti-competitive practice employed by Amazon, e.g., prioritizing search results to showcase its own brand products, further aggravates the situation facing the sellers (Wall Street Journal, 2019). Consequently, politicians and regulators have proposed a certain policy of prohibiting platforms from introducing the own brand products in order to protect
the incumbent sellers.
This study addresses two research questions of critical interest to both the policy makers and the incumbent sellers. First, is the platform's introducing its own brand product always detrimental to the incumbent sellers? Second, how effective is the proposed policy of prohibiting the platform from its own brand introduction in terms of protecting the incumbent sellers? We build a stylized model to examine the impact of the platform's own brand introduction on the incumbent sellers under two prevailing pricing contracts: sell-on (i.e., the agency model) and sell-to contract (i.e., the traditional wholesale model). We first analyze the status quo where the platform introduces its own brand and there is no regulation on the platform, followed by analyses of the introduction of a new brand as opposed to the own brand by the platform. We next examine the platform's incentive with regard to whether to introduce its own brand and its preferences for the type of pricing contract. Finally, we dissect the platform's strategic response to each of the three possible policies of barring the platform's own brand product, and evaluate the effectiveness of each policy accordingly.

In the absence of regulation, the platform's decisions on introducing an own brand and its choices on pricing contracts always hurt the sellers. Furthermore, the proposed legislation "that prohibits platforms from both offering a marketplace for commerce and participating in that marketplace" by Senator Warren does not have the desired outcome of helping the incumbent sellers. Instead, it forces the platform to adopt only the sell-to contract with own brand introduction that hurts the sellers under most market conditions. Interestingly, when the own brand introduction is banned under the sell-to contract, the incumbent sellers can be better off because the platform's strategic reaction to the enforcement can lead to the best scenario for the incumbent sellers. If the ban is imposed on both the sell-on and sell-to contracts, the platform's best response is to add another new brand competing with the incumbent sellers, which can also help the incumbent sellers, however, not as much as in the case of the enforcement only under the sell-to contract.

This study has several limitations that present future research opportunities. First, our baseline model assumes that the commission rate is exogenous regardless of the platform's brand launch in the sell-on contract because in practice, the platform's commission rate remains stable. Future study can further explore the platform's strategic decision on the commission rate, jointly with its
brand introduction. Second, we do not consider the platform's information advantage for its own brand product design or its intervention on the incumbents' market competition. This is because the objective of this research is to examine the effect of the platform's own brand introduction on incumbent sellers under different pricing contracts, and more importantly, to assess the effectiveness of the policy that prohibits the platform's own brand in helping the incumbents by considering the platform's strategic decision on the brand introduction and pricing contracts. Future research may focus more on how the platform can utilize its information advantage and its optimization problem in terms of its product or market design. Third, while we discuss the platform's incentives and policy implications on its own brand or a new brand across different pricing contracts, we do not consider the strategic choice of the base level demand. This setup helps us focus more on the meaningful comparisons among different scenarios and pricing contracts. Future study can further incorporate a cost component into the platform's profit function a nd generate the optimal level of base level demand in different cases. Notice that our key message urging to carefully implement the ban is driven by (i) the nature of the new entry that changes competition in the existing market, (ii) the advantage of the platform which can mold market outcome to a large extent for its own profit gain, and (iii) the difference between the two contracts. We demonstrate that our key messages and insights continue to hold in the utility-driven demand models. However, we also acknowledge that this study examines the equilibrium of interior solutions where all firms have a positive market share and compete with all others in each scenario to focus on the policy effectiveness. Future study can complete understanding of the platform's and sellers' strategic moves by exploring their entire paths. Lastly, we follow the prior literature on the platform's brand introduction and normalize the production cost to zero. It will be of interest for future research to study the impact of the platform's innovation cost for its brand introduction.

## References

Abhishek, Vibhanshu, Kinshuk Jerath, John Zhang. 2016. Agency selling or reselling? Channel structures in electronic retailing. Management Science 62(8) 2259-2280.

Bloomberg, Noah Smith. 2019. Amazon's winner-take-all approach to small business. Tech. rep., Bloomberg.

Boyaci, Tamer, Guillermo Gallego. 2004. Supply chain coordination in a market with customer service competition. Production and Operations Management 13(1) 3-22.

Business Insider, Connie Chen. 2018. Amazon now sells 76 of its own private-label brands from clothes to baby wipes. Tech. rep., Business insider.

Business Insider, Shona Ghosh. 2017a. Amazon's private brands have gone from $\$ 0$ to $\$ 2.5$ billion in sales in less than a decade. Tech. rep., Business insider.

Business Insider, Tanya Dua. 2017b. There's one app millennials can't live without, and it's great news for Jeff Bezos. Tech. rep., Business Insider.

Cachon, Gérard P, A Gürhan Kök. 2010. Competing manufacturers in a retail supply chain: On contractual form and coordination. Management Science 56(3) 571-589.

Cachon, Gérard P, Martin A Lariviere. 2005. Supply chain coordination with revenue-sharing contracts: Strengths and limitations. Management Science 51(1) 30-44.

Chen, Jianqing, Zhiling Guo. 2014. Online advertising, retailer platform openness, and long tail sellers. Working paper, The University of Texas at Dallas.

Chen, Yongmin, Michael H. Riordan. 2007. Price and variety in the spokes model. The Economic Journal 117(522) 897-921.

Chintagunta, Pradeep K, Andre Bonfrer, Inseong Song. 2002. Investigating the effects of store-brand introduction on retailer demand and pricing behavior. Management Science 48(10) 1242-1267.

CNBC, Eugene Kim. 2018. Amazon is doubling down on its private label business, stoking 'huge fear' in some sellers. Tech. rep., CNBC.

CNBC, Lauren Feiner. 2019. Amazon admits to congress that it uses 'aggregated' data from thirdparty sellers to come up with its own products. Tech. rep., CNBC.

Coughlan, Anne T. 1985. Competition and cooperation in marketing channel choice: Theory and application. Marketing Science 4(2) 110-129.

Dhar, Sanjay K, Stephen J Hoch. 1997. Why store brand penetration varies by retailer. Marketing Science 16(3) 208-227.

Forbes, Pamela N. Danziger. 2018. Thinking of selling on amazon marketplace? Here are the pros and cons. Tech. rep., Forbes.

Gupta, Sudheer, Richard Loulou. 1998. Process innovation, product differentiation, and channel structure: Strategic incentives in a duopoly. Marketing Science 17(4) 301-316.

Hagiu, Andrei, Julian Wright. 2015. Marketplace or reseller? Management Science 61(1) 184-203.
Hao, Lin, Ming Fan. 2014. An analysis of pricing models in the electronic book market. MIS Quarterly 38(4) 1017-1032.

Huang, He, Geoffrey Parker, Yinliang Ricky Tan, Hongyan Xu. 2020. Altruism or shrewd business? Implications of technology openness on innovations and competition. MIS Quarterly 44(3) 10491071.

Internet Retailer Research, Channel Advisor. 2017. The Amazon report 2017-global leader in eCommerce data. Tech. rep., Internet-Retailer Research-Vertical Web Media.

Internet Retailer Research, Katie Evans. 2018. Amazon says more than 1 million U.S. small and medium-sized businesses sell on its marketplace. Tech. rep., Internet Retailer Research.

Iyer, Ganesh, David Soberman, J. Miguel Villas-Boas. 2005. The targeting of advertising. Marketing Science 24(3) 461-476.

Jiang, B., K. Jerath, K. Srinivasan. 2011. Firm strategies in the "mid tail" of platform-based retailing. Marketing Science 30(5) 757-775.

Kumar, Nirmalya, Jan-Benedict E. M. Steenkamp. 2007. Private label strategy: How to meet the store brand challenge. Harvard Business School Press.

Kwark, Young, Jianqing Chen, Srinivasan Raghunathan. 2014. Online product reviews: Implications for retailers and competing manufacturers. Information Systems Research 25(1) 93-110.

Lee, Eunkyu, Richard Staelin. 2000. A general theory of demand in a multi-product multi-outlet market. Working paper.

Mantin, Benny, Harish Krishnan, Tirtha Dhar. 2014. The strategic role of third-party marketplaces in retailing. Production and Operations Management 23(11) 1937-1949.

McGuire, Timothy W., Richard Staelin. 1983. An industry equilibrium analysis of downstream vertical integration. Marketing Science 2(2) 161-191.

Mills, David E. 1999. Private labels and manufacturer counterstrategies. European Review of Agricultural Economics 26(2) 125-145.

Moorthy, K. Sridhar. 1988. Strategic decentralization in channels. Marketing Science 7(4) 335-355.
Morton, Fiona Scott, Florian Zettelmeyer. 2004. The strategic positioning of store brands in retailermanufacturer negotiations. Review of Industrial Organization 24(2) 161-194.

Nasser, Sherif, Danko Turcic, Chakravarthi Narasimhan. 2013. National brand's response to store brands: Throw in the towel or fight back? Marketing Science 32(4) 591-608.

Raju, J.S, R. Sethuraman, S.K Dhar. 1995. The introduction and performance of store brands. Management Science 41(6) 957-978.

Salop, Steven C. 1979. Monopolistic competition with outside goods. The Bell Journal of Economics 141-156.

Sayman, S., S.J Hoch, J.S Raju. 2002. Positioning of store brands. Marketing Science 21(4) 378-397.
Sayman, S., J.S Raju. 2004. How category characteristics affect the number of store brands offered by the retailer: A model and empirical analysis. Journal of Retailing 80(4) 279-287.

Shubik, Martin, Richard Levitan. 1980. Market Structure and Behavior. Harvard University Press.
Statista, J. Clement. 2019. Retail e-commerce sales in the United States from 2017 to 2023. Tech. rep., Statista.

Sun, C., B. Kofal, Y. Ji, R. Patterson. 2017. Business-to-consumer platform strategy: How vendor certification changes platform and seller incentives. ACM Transactions on Management Information Systems 8(2-3) 6.

Tan, Yinliang, Janice E. Carrillo, Hsing Kenny Cheng. 2016. The agency model for digital goods. Decision Sciences 47(4) 628-660.

The Capital Forum, Antitrust Enforcement. 2016. Amazon: By prioritizing its own fashion label brands in product placement on its increasingly dominant platform. Tech. rep., The Capital Forum.

The Harris Poll, Reputation Quotient Summary Report. 2018. The reputations of the most visible companies. Tech. rep., The Harris Poll.

The New York Times, Adam Satariano. 2019. Amazon faces E.U. inquiry over data from independent sellers .

Thomson, James, Joseph Hansen. 2016. The Amazon marketplace dilemma: A brand executive's challenge growing sales and maintaining control. Tech. rep., Buy box Experts.

Wall Street Journal, Dana Mattioli. 2019. Amazon changed search algorithm in ways that boost its own products. Tech. rep., Wall Street Journal.

Wu, Hao, Gangshu (George) Cai, Jian Chen, Chwen Sheu. 2015. Online manufacturer referral to heterogeneous retailers. Production and Operations Management 24(11) 1768-1782.

Zhu, Feng, Qihong Liu. 2018. Competing with complementors: An empirical look at Amazon.com. Strategic Management Journal 39(10) 2618-2642.

# Online Appendix to "Impact of Own Brand Product Introduction on Optimal Pricing Models for Platform and Incumbent Sellers" 

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## A1 Proof of Proposition 1

Proof: We analyze the impact of own brand on independent sellers by comparing sellers' profit in two scenarios under the sell-on contract. We first show the equilibrium outcomes at both benchmark and own brand scenario, and then compare the sellers' profit in these two scenarios. First, in the benchmark scenario, we take the demand functions in Equation (1) into the profit functions in Equation (3). Two sellers' optimization problems in stage 1 are characterized by solving the first-order condition of their profits:

$$
\begin{aligned}
& \frac{\partial \pi_{1}^{s}}{\partial p_{1}^{s}}=\frac{1}{2}(1-\alpha)\left(1-2(1+\theta) p_{1}^{s}+\theta p_{2}^{s}\right)=0, \\
& \frac{\partial \pi_{2}^{s}}{\partial p_{2}^{s}}=\frac{1}{2}(1-\alpha)\left(1-2(1+\theta) p_{2}^{s}+\theta p_{1}^{s}\right)=0,
\end{aligned}
$$

from which we can derive the optimal retail price of product $i, i \in\{1,2\} \mathrm{s}$ :

$$
p_{i}^{s}=\frac{1}{2+\theta} .
$$

Substituting the retail prices, we can obtain the sellers' and platform's equilibrium profit:

$$
\begin{align*}
& \pi_{i}^{s}=(1-\alpha) p_{i}^{s} D_{i}^{s}=\frac{(1+\theta)(1-\alpha)}{2(2+\theta)^{2}}, \quad \text { for } i=\{1,2\}  \tag{19}\\
& \pi_{r}^{s}=\alpha\left(p_{1}^{s} D_{1}^{s}+p_{2}^{s} D_{2}^{s}\right)=\frac{(1+\theta) \alpha}{(2+\theta)^{2}} .
\end{align*}
$$

Second, in the own brand scenario, we take the demand functions in Equation (2) into the sellers' profit functions in Equation (5) and the platform's profit function in Equation (4). The optimization problems of two sellers and a platform in stage 1 are characterized by solving the first-order condition of their profits:

$$
\begin{aligned}
& \frac{\partial \pi_{1}^{o}}{\partial p_{1}^{o}}=\frac{(1-\alpha)\left(2-2 p_{1}^{o}(2+\delta+\theta)+\theta p_{2}^{o}+\delta p_{r}^{o}\right)}{2(2+a)}=0 \\
& \frac{\partial \pi_{2}^{o}}{\partial p_{2}^{o}}=\frac{(1-\alpha)\left(2-2 p_{2}^{o}(2+\delta+\theta)+\theta p_{1}^{o}+\delta p_{r}^{o}\right)}{2(2+a)}=0 \\
& \frac{\partial \pi_{r}^{o}}{\partial p_{r}^{o}}=\frac{2 a+\delta p_{1}^{o}(1+\alpha)+\delta p_{2}^{o}(1+\alpha)-4(1+\delta) p_{r}^{o}}{2(2+a)}=0
\end{aligned}
$$

from which we can derive the optimal retail prices for sellers and platform:

$$
\begin{aligned}
& p_{i}^{o}=\frac{(4+a) \delta+4}{\delta^{2}(3-\alpha)+2 \delta(6+\theta)+2(4+\theta)} \quad \text { for } i=\{1,2\} \\
& p_{r}^{o}=\frac{a(4+2 \delta+\theta)+2 \delta(1+\alpha)}{\delta^{2}(3-\alpha)+2 \delta(6+\theta)+2(4+\theta)}
\end{aligned}
$$

Substituting the retail prices, we can characterize the sellers' and platform's equilibrium profit:

$$
\begin{align*}
\pi_{i}^{o}=(1-\alpha) p_{i}^{o} D_{i}^{o}= & \frac{(1-\alpha)((4+a) \delta+4)^{2}(2+\delta+\theta)}{2(2+a)\left(\delta^{2}(3-\alpha)+2 \delta(6+\theta)+2(4+\theta)\right)^{2}}, \quad \text { for } i=\{1,2\} \\
\pi_{r}^{o}=p_{r}^{o} D_{r}^{o}+\sum_{i=1}^{2} \alpha p_{i}^{o} D_{i}^{o}= & \frac{(a(\delta((2-\alpha) \delta+\theta+6)+\theta+4)+2(1-\alpha) \delta(\delta+1))(a(2 \delta+\theta+4)+2(\alpha+1) \delta)}{(a+2)\left((3-\alpha) \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2}}  \tag{20}\\
& +\frac{\alpha((a+4) \delta+4)^{2}(\delta+\theta+2)}{(a+2)\left((3-\alpha) \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2}}
\end{align*}
$$

In the sell-on contract, using sellers' profit in own brand scenario (i.e., $\pi_{i}^{o}$ in Equation (20)) and their profit in benchmark scenario (i.e., $\pi_{i}^{s}$ in Equation (19)), we can get $\pi_{i}^{o}>\pi_{i}^{s}, i \in\{1,2\}$ if and only if (i) $a<a_{11}$ and $\delta<\delta_{1}$ or (ii) $a>a_{12}$. Note that

$$
\pi_{i}^{o}-\pi_{i}^{s}=\frac{(1-\alpha)\left(A a^{2}+B a+C\right)}{2(a+2)(\theta+2)^{2}\left[\delta^{2}(3-\alpha)+2 \delta(\theta+6)+2(\theta+4)\right]^{2}}
$$

where $A=\delta^{2}(\theta+2)^{2}(\delta+\theta+2)>0$;
$B=\delta^{4}(-(\theta+1))(\alpha-3)^{2}-4 \delta^{3}\left(\theta^{2}-(\theta+1)(\theta+6) \alpha+13 \theta+10\right)+4 \delta^{2}(\theta(\theta(\theta+\alpha-2)+5 \alpha-31)+4(\alpha-6))-8 \delta(\theta(5 \theta+22)+$ 16) $-4(\theta+1)(\theta+4)^{2}<0$;
$C=2 \delta^{4}(\theta+1)(3-\alpha)^{2}-8 \delta^{3}\left(\theta^{2}-(\theta+1)(\theta+6) \alpha+13 \theta+10\right)-8 \delta^{2}\left(-\theta^{3}-(\theta+1)(\theta+4) \alpha+23 \theta+16\right)-16 \delta(2-\theta)(\theta(\theta+$ 4) +2$)+8 \theta^{2}(\theta+3)$.

We can verify that this quadratic function of $A a^{2}+B a+C$ in $a$ has two real solutions (i.e., $\left.B^{2}-4 A C>0\right)$. We denote these two solutions of $a$, which leads to $A a^{2}+B a+C=0$, as $a_{11}$ and $a_{12}$, where $a_{11}=\frac{-B-\sqrt{B^{2}-4 A C}}{2 A}$, and $a_{12}=\frac{-B+\sqrt{B^{2}-4 A C}}{2 A}$. The base level demand $a$ should be non-negative. We can easily show that $a_{12}>0$ always holds, and $a_{11}>0$ if only if $\delta<\delta_{1}$, where $\delta_{1}$ is the only solution that is in the range of $(0,1)$ for $-B-\sqrt{B^{2}-4 A C}=0$. Therefore, $A a^{2}+B a+C>0$ if and only if (i) $a<a_{11}$ and $\delta<\delta_{1}$ or (ii) $a>a_{12}$. In other words, $\pi_{i}^{o}>\pi_{i}^{s}$, $i \in\{1,2\}$ if and only if (i) $a<a_{11}$ and $\delta<\delta_{1}$ or (ii) $a>a_{12}$.

## A2 Proof of Proposition 2

Proof: We analyze the impact of own brand on independent sellers by comparing sellers' profit in two scenarios under the sell-to contract. We first show the equilibrium outcome at both benchmark
and own brand scenario, and then compare the sellers' profit in these scenarios. In the benchmark scenario, we take the demand functions in Equation (1) into the platform's profit in Equation (6). The platform's optimization problem in stage 2 is characterized by the first-order conditions of Equation (6):

$$
\begin{aligned}
& \frac{\partial \pi_{r}^{s}}{\partial p_{1}^{s}}=\frac{1}{2}\left(1-2(\theta+1) p_{1}^{s}+2 \theta p_{2}^{s}+(\theta+1) w_{1}^{s}-\theta w_{2}^{s}\right)=0 \\
& \frac{\partial \pi_{r}^{s}}{\partial p_{2}^{s}}=\frac{1}{2}\left(1-2(\theta+1) p_{2}^{s}+2 \theta p_{1}^{s}+(\theta+1) w_{2}^{s}-\theta w_{1}^{s}\right)=0
\end{aligned}
$$

from which we can derive the optimal retail price of product $i, i \in\{1,2\}$ as a function of the wholesale price, $p_{i}^{s}=\frac{1}{2}\left(w_{i}^{s}+1\right)$. The sellers' optimization problem in stage 1 is characterized by the first-order condition of Equation (7):

$$
\begin{aligned}
& \frac{\partial \pi_{1}^{s}}{\partial w_{1}^{s}}=\frac{1}{2}\left(1+\theta p_{2}^{s}-(\theta+1) p_{1}^{s}\right)=\frac{1}{4}\left(1-2(\theta+1) w_{1}^{s}+\theta w_{2}^{s}\right)=0 \\
& \frac{\partial \pi_{2}^{s}}{\partial w_{2}^{s}}=\frac{1}{2}\left(1+\theta p_{1}^{s}-(\theta+1) p_{2}^{s}\right)=\frac{1}{4}\left(1-2(\theta+1) w_{2}^{s}+\theta w_{1}^{s}\right)=0
\end{aligned}
$$

from which we can derive the optimal wholesale price of product $i, i \in\{1,2\}$ :

$$
w_{i}^{s}=\frac{1}{2+\theta}
$$

Substituting the above optimal wholesale price into the above optimal retail price function of product $i, i \in\{1,2\}$ we derive the optimal retail price:

$$
p_{i}^{s}=\frac{\theta+3}{2 \theta+4}
$$

With the above equilibrium wholesale and retail price, demand functions in Equation (1), platform's profit function in Equation (6), and sellers' profit function in Equation (7), we have equilibrium profits:

$$
\begin{array}{ll}
\pi_{i}^{s}=w_{i}^{s} D_{i}^{s} & =\frac{(1+\theta)}{4(2+\theta)^{2}}, \quad \text { for } i=\{1,2\} \\
\pi_{r}^{s}=\sum_{i=1}^{2}\left(p_{i}^{o}-w_{i}^{o}\right) D_{i}^{o}=\frac{(1+\theta)^{2}}{4(2+\theta)^{2}} . \tag{21}
\end{array}
$$

In the own brand scenario, we take the demand functions in Equation (2) into the platform's profit functions in Equation (8). The platform's optimization problem in stage 2 is characterized
by the first-order conditions of Equation (8):

$$
\begin{aligned}
& \frac{\partial \pi_{r}^{o}}{\partial p_{1}^{o}}=\frac{2-2 p_{1}^{o}(\delta+\theta+2)+2 \delta p_{r}^{o}+\theta\left(2 p_{2}^{o}-w_{2}^{o}\right)+w_{1}^{o}(\delta+\theta+2)}{2(a+2)}=0 \\
& \frac{\partial \pi_{r}^{o}}{\partial p_{2}^{o}}=\frac{2-2 p_{2}^{o}(\delta+\theta+2)+2 \delta p_{r}^{o}+\theta\left(2 p_{1}^{o}-w_{1}^{o}\right)+w_{2}^{o}(\delta+\theta+2)}{2(a+2)}=0 \\
& \frac{\partial \pi_{r}^{o}}{\partial p_{r}^{o}}=\frac{2 a-4(\delta+1) p_{r}^{o}+\delta\left(2 p_{1}^{o}+2 p_{2}^{o}-w_{1}^{o}-w_{2}^{o}\right)}{2(a+2)}=0
\end{aligned}
$$

from which we can derive the platform's optimal retail prices as functions of the wholesale price: $p_{i}^{o}=\frac{(a+2) \delta+(3 \delta+2) w_{i}^{o}+2}{6 \delta+4}$ for $i=\{1,2\}$ and $p_{r}^{o}=\frac{a(\delta+2)+2 \delta}{6 \delta+4}$. The sellers' optimization problem in stage 1 is characterized by the first-order condition of Equation (9):

$$
\begin{aligned}
& \frac{\partial \pi_{1}^{o}}{\partial w_{1}^{o}}=\frac{2-(\delta+\theta+2) p_{1}^{o}+\theta p_{2}^{o}+\delta p_{s}^{o}}{2(a+2)}=\frac{2-2 w_{1}^{o}(\delta+\theta+2)+\theta w_{2}^{o}}{4(a+2)}=0 \\
& \frac{\partial \pi_{2}^{o}}{\partial w_{2}^{o}}=\frac{2-(\delta+\theta+2) p_{2}^{o}+\theta p_{1}^{o}+\delta p_{s}^{o}}{2(a+2)}=\frac{2-2 w_{2}^{o}(\delta+\theta+2)+\theta w_{1}^{o}}{4(a+2)}=0
\end{aligned}
$$

from which we can derive the optimal wholesale price of product $i, i \in\{1,2\}$

$$
w_{i}^{o}=\frac{2}{2 \delta+\theta+4} .
$$

Substituting the above optimal wholesale price into platform's optimal retail price functions, we derive the optimal retail prices:

$$
\begin{aligned}
& p_{i}^{o}=\frac{(a+2) \delta+2}{6 \delta+4}+\frac{1}{2 \delta+\theta+4}, \quad \text { for } i=\{1,2\} \\
& p_{r}^{o}=\frac{a(\delta+2)+2 \delta}{6 \delta+4}
\end{aligned}
$$

With the above equilibrium wholesale and retail prices, demand functions in Equation (2), platform's profit function in Equation (8), and sellers' profit function in Equation (9), we have the equilibrium profits:

$$
\begin{array}{cll}
\pi_{i}^{o}=w_{i}^{o} D_{i}^{o} & =\frac{\delta+\theta+2}{(a+2)(\theta \delta+\theta+4)^{2}}, & \text { for } i=\{1,2\} \\
\pi_{r}^{o}=p_{r}^{o} D_{r}^{o}+\sum_{i=1}^{2}\left(p_{i}^{o}-w_{i}^{o}\right) D_{i}^{o} & =\frac{2\left(a^{2}+2\right)+(a+2)^{2} \delta}{4(a+2)(3 \delta+2)}-\frac{3 \delta+2 \theta+6}{(a+2)(2 \delta+\theta+4)^{2}} & \tag{22}
\end{array}
$$

In the sell-to contract, using sellers' profit in own brand scenario (i.e., $\pi_{i}^{o}$ in Equation (22)) and their profit in benchmark scenario (i.e., $\pi_{i}^{s}$ in Equation (21)), we can get $\pi_{i}^{o}>\pi_{i}^{s}$ if and only if $a<a_{2}$ and $\delta<\delta_{2}, i \in\{1,2\}$ because

$$
\pi_{i}^{o}-\pi_{i}^{s}=\frac{-a(\theta+1)(2 \delta+\theta+4)^{2}-8 \delta^{2}(\theta+1)+4 \delta\left(\theta^{2}+6 \theta+4\right)+2 \theta^{2}(\theta+3)}{4(a+2)(\theta+2)^{2}(2 \delta+\theta+4)^{2}}
$$

which is positive if and only if $a<\frac{-8 \delta^{2}(\theta+1)-4 \delta(\theta(\theta+6)+4)+2 \theta^{2}(\theta+3)}{(\theta+1)(2 \delta+\theta+4)^{2}}:=a_{2}$. The threshold $a_{2}$ is positive if and only if $\delta<\frac{-\theta(\theta+6)+\sqrt{(\theta+2)^{2}(\theta(5 \theta+8)+4)}-4}{4(\theta+1)}:=\delta_{2}$.

## A3 Proof of Proposition 3

Proof: To comprehensively analyze the effectiveness of removing own brand under the sell-on contract, we also consider a scenario where the platform introduces a new brand. We first show the equilibrium at this new brand scenario under the sell-on contract, and then compare the sellers' profit in this scenario with benchmark and own brand scenarios. In the new-brand scenario, we take the demand functions $D_{n}^{n}$, which is analogous to $D_{r}^{o}$ in Equation (2), and $D_{i}^{n}$, which is analogous to $D_{i}^{o}$ in Equation (2), into the profit functions in Equations (10) and (11), respectively. Three sellers' optimization problems in stage 1 are characterized by solving the first-order condition of their profits:

$$
\begin{aligned}
& \frac{\partial \pi_{1}^{n}}{\partial p_{1}^{n}}=\frac{(1-\alpha)\left(2-2 p_{1}^{n}(\delta+\theta+2)+\theta p_{2}^{n}+\delta p_{n}^{n}\right)}{2(a+2)}=0 \\
& \frac{\partial \pi_{2}^{n}}{\partial p_{2}^{n}}=\frac{(1-\alpha)\left(2-2 p_{2}^{n}(\delta+\theta+2)+\theta p_{1}^{n}+\delta p_{n}^{n}\right)}{2(a+2)}=0 \\
& \frac{\partial \pi_{n}^{n}}{\partial p_{n}^{n}}=\frac{(1-\alpha)\left(2 a+\delta p_{1}^{n}+\delta p_{2}^{n}-4(\delta+1) p_{n}^{n}\right)}{2(a+2)}=0
\end{aligned}
$$

from which we can derive the optimal retail prices

$$
\begin{aligned}
p_{i}^{n} & =\frac{(a+4) \delta+4}{3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)} \quad \text { for } i=\{1,2\} \\
p_{n}^{n} & =\frac{a(2 \delta+\theta+4)+2 \delta}{3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)}
\end{aligned}
$$

Substituting the retail prices, we can characterize the sellers' and platform's equilibrium profit:

$$
\begin{align*}
\pi_{i}^{n}=(1-\alpha) p_{i}^{n} D_{i}^{n} & =\frac{(1-\alpha)((a+4) \delta+4)^{2}(\delta+\theta+2)}{2(a+2)\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2}}, \quad \text { for } i=\{1,2\} \\
\pi_{n}^{n}=(1-\alpha) p_{n}^{n} D_{n}^{n} & =\frac{(1-\alpha)(\delta+1)(a(2 \delta+\theta+4)+2 \delta)^{2}}{(a+2)\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2}}  \tag{23}\\
\pi_{r}^{n}=\alpha p_{n}^{n} D_{n}^{n}+\sum_{i=1}^{2} \alpha p_{i}^{n} D_{i}^{n} & =\frac{\alpha(\delta+1)(2(a+1) \delta+a(\theta+4))^{2}}{(a+2)\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2}}+\frac{\alpha((a+4) \delta+4)^{2}(\delta+\theta+2)}{(a+2)\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2}}
\end{align*}
$$

In the sell-on contract, using the sellers' profit in benchmark scenario (i.e., $\pi_{i}^{s}$ in Equation (19)), in own brand scenario (i.e., $\pi_{i}^{o}$ in Equation (20)), and in new-brand scenario (i.e., $\pi_{i}^{n}$ in Equa-
tion (23)), we have the following results:
(a) $\pi_{i}^{n}>\pi_{i}^{s}, i \in\{1,2\}$ if and only if (i) $a<a_{31}$ and $\delta<\delta_{3}$ or (ii) $a>a_{32}$. Note that

$$
\pi_{i}^{n}-\pi_{i}^{s}=\frac{(1-\alpha)\left(A a^{2}+B a+C\right)}{2(a+2)(\theta+2)^{2}\left[\delta^{2}(3-\alpha)+2 \delta(\theta+6)+2(\theta+4)\right]^{2}}
$$

where $A=\delta^{2}(\theta+2)^{2}(\delta+\theta+2)>0$;
$B=-9 \delta^{4}(\theta+1)-4 \delta^{3}(\theta(\theta+13)+10)+4 \delta^{2}(\theta((\theta-2) \theta-31)-24)-8 \delta(\theta(5 \theta+22)+16)-4(\theta+1)(\theta+4)^{2}<0 ;$
$C=-18 \delta^{4}(\theta+1)-8 \delta^{3}(\theta(\theta+13)+10)+8 \delta^{2}\left(\theta^{3}-23 \theta-16\right)+16 \delta(\theta-2)(\theta(\theta+4)+2)+8 \theta^{2}(\theta+3)$.
We can verify that this quadratic function of $A a^{2}+B a+C$ in $a$ has two real solutions (i.e., $\left.B^{2}-4 A C>0\right)$. We denote these two solutions of $a$, which leads to $A a^{2}+B a+C=0$, as $a_{31}$ and $a_{32}$, where $a_{31}=\frac{-B-\sqrt{B^{2}-4 A C}}{2 A}$, and $a_{32}=\frac{-B+\sqrt{B^{2}-4 A C}}{2 A}$. The base level demand $a$ should be non-negative. We can easily show that $a_{32}>0$ always holds, and $a_{31}>0$ if only if $\delta<\delta_{3}$, where $\delta_{3}$ is the only solution that is in the range of $(0,1)$ for $-B+\sqrt{B^{2}-4 A C}=0$. Therefore, $A a^{2}+B a+C>0$ if and only if (i) $a<a_{31}$ and $\delta<\delta_{3}$ or (ii) $a>a_{32}$. In other words, $\pi_{i}^{n}>\pi_{i}^{s}$, $i \in\{1,2\}$ if and only if (i) $a<a_{31}$ and $\delta<\delta_{3}$ or (ii) $a>a_{32}$.
(b) $\pi_{i}^{o}>\pi_{i}^{n}$ because $\pi_{i}^{o}-\pi_{i}^{n}=\frac{\delta^{2}(1-\alpha) \alpha((a+4) \delta+4)^{2}(\delta+\theta+2)\left(\delta^{2}(6-\alpha)+4 \delta(\theta+6)+4(\theta+4)\right)}{2(a+2)\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2}\left(\delta^{2}(3-\alpha)+2 \delta(\theta+6)+2(\theta+4)\right)^{2}}>0$ always holds.

## A4 Proof of Proposition 4 and Corollary 1

Proof: To comprehensively analyze the effectiveness of removing own brand under the sell-to contract, we also consider a scenario where the platform introduces a new brand. We first show the equilibrium at this new brand scenario under the sell-to contract, and then compare the sellers' profit in this scenario with benchmark and own brand scenarios. In the new-brand scenario, we take the demand functions $D_{n}^{n}$, which is analogous to $D_{r}^{o}$ in Equation (2), and $D_{i}^{n}$, which is analogous to $D_{i}^{o}$ in Equation (2), into the platform's profit functions in Equation (12). The platform's optimization problem in stage 2 is characterized by the first-order conditions of Equation (12):

$$
\begin{aligned}
& \frac{\partial \pi_{r}^{n}}{\partial p_{1}^{n}}=\frac{2-2 p_{1}^{n}(\delta+\theta+2)+2 \delta p_{n}^{n}+\theta\left(2 p_{2}^{n}-w_{2}^{n}\right)-\delta w_{n}^{n}+w_{1}^{n}(\delta+\theta+2)}{2(a+2)}=0 \\
& \frac{\partial \pi_{r}^{n}}{\partial p_{2}^{n}}=\frac{2-2 p_{2}^{n}(\delta+\theta+2)+2 \delta p_{n}^{n}+\theta\left(2 p_{1}^{n}-w_{1}^{n}\right)-\delta w_{n}^{n}+w_{2}^{n}(\delta+\theta+2)}{2(a+2)}=0 \\
& \frac{\partial \pi_{r}^{n}}{\partial p_{n}^{n}}=\frac{2\left(a-2 p_{n}^{n}+w_{n}^{n}\right)+\delta\left(-4 p_{n}^{n}+2 p_{1}^{n}+2 p_{2}^{n}+2 w_{n}^{n}-w_{1}^{n}-w_{2}^{n}\right)}{2(a+2)}=0
\end{aligned}
$$

from which we can derive the optimal retail prices as a function of the wholesale price: $p_{i}^{n}=$ $\frac{(a+2) \delta+(3 \delta+2) w_{i}^{n}+2}{6 \delta+4}$ for $i=\{1,2\}$ and $p_{n}^{n}=\frac{a(\delta+2)+2 \delta+(3 \delta+2) w_{n}^{n}}{6 \delta+4}$. The sellers' optimization problem in stage 1 is characterized by the first-order condition of Equations (13) and (14):

$$
\begin{aligned}
& \frac{\partial \pi_{1}^{n}}{\partial w_{1}^{n}}=\frac{2-(\delta+\theta+2) p_{1}^{n}+\theta p_{2}^{n}+\delta p_{n}^{n}}{2(a+2)}=\frac{2-2 w_{1}^{n}(\delta+\theta+2)+\theta w_{2}^{n}+\delta w_{n}^{n}}{4(a+2)}=0 \\
& \frac{\partial \pi_{2}^{n}}{\partial w_{2}^{n}}=\frac{2-(\delta+\theta+2) p_{2}^{n}+\theta p_{1}^{n}+\delta p_{n}^{n}}{2(a+2)}=\frac{2-2 w_{2}^{n}(\delta+\theta+2)+\theta w_{1}^{n}+\delta w_{n}^{n}}{4(a+2)}=0 \\
& \frac{\partial \pi_{n}^{n}}{\partial w_{n}^{n}}=\frac{2 a+\delta p_{1}^{n}+\delta p_{2}^{n}-2(\delta+1) p_{n}^{n}}{2 a+4}=\frac{2 a-4(\delta+1) w_{n}^{n}+\delta w_{1}^{n}+\delta w_{2}^{n}}{4(a+2)}=0
\end{aligned}
$$

from which we can derive the optimal wholesale price:

$$
\begin{aligned}
& w_{i}^{n}=\frac{(a+4) \delta+4}{3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)}, \text { for } i=\{1,2\}, \\
& w_{n}^{n}=\frac{a(2 \delta+\theta+4)+2 \delta}{3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)} .
\end{aligned}
$$

Substituting the above optimal wholesale price into platform's optimal retail price functions, we derive the optimal retail prices:

$$
\begin{array}{ll}
p_{i}^{n}=\frac{1}{2}\left(\frac{(a+4) \delta+4}{3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)}+\frac{(a+2) \delta+2}{3 \delta+2}\right), & \text { for } i=\{1,2\}, \\
p_{n}^{n}=\frac{\delta((a+4) \delta+4)}{4(\delta+1)\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)}+\frac{a\left(2 \delta^{2}+9 \delta+6\right)+4 \delta(\delta+1)}{4(\delta+1)(3 \delta+2)} . &
\end{array}
$$

With the above equilibrium wholesale and retail price, demand functions, platform's profit function in Equation (12), and sellers' profit function in Equation (13) and (14), we have equilibrium profits:

$$
\begin{align*}
\pi_{i}^{n}=w_{i}^{n} D_{i}^{n} & =\frac{((a+4) \delta+4)^{2}(\delta+\theta+2)}{4(a+2)\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2}}, \quad \text { for } i=\{1,2\}, \\
\pi_{n}^{n}=w_{n}^{n} D_{n}^{n}= & \frac{(\delta+1)(a(2 \delta+\theta+4)+2 \delta)^{2}}{2(a+2)\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2}}, \\
\pi_{r}^{n}=\left(p_{n}^{n}-w_{n}^{n}\right) D_{n}^{n}+\sum_{i=1}^{2}\left(p_{i}^{n}-w_{i}^{n}\right) D_{i}^{n}= & \frac{4(\delta+1) \theta(a \delta+\delta+1)+3 \delta(a(\delta(2 \delta+3)+2)+2 \delta(\delta+1))}{2(a+2)(3 \delta+2)\left(3 \delta^{2}+2 \delta \theta+12 \delta+2 \theta+8\right)}  \tag{24}\\
& +\frac{a^{2}(\delta(\delta(\delta(12 \delta+8 \theta+39)+14 \theta+62)+10 \theta+48)+4(\theta+4))}{16(a+2)(\delta+1)(3 \delta+2)\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)} \\
& +\frac{(\delta(\delta+12)+8)((a+4) \delta+4)^{2}}{16(a+2)(\delta+1)\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2}}
\end{align*}
$$

In the sell-to contract, using the sellers' profit in benchmark scenario (i.e., $\pi_{i}^{s}$ in Equation (21)), in own brand scenario (i.e., $\pi_{i}^{o}$ in Equation (22)), and in new-brand scenario (i.e., $\pi_{i}^{n}$ in Equation (24)), we have the following results:
(a) $\pi_{i}^{n}>\pi_{i}^{s}, i \in\{1,2\}$ if and only if (i) $a<a_{31}$ and $\delta<\delta_{3}$ or (ii) $a>a_{32}$ because

$$
\pi_{i}^{n}-\pi_{i}^{s}=\frac{A a^{2}+B a+C}{4(a+2)\left[3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right]^{2}(\theta+2)^{2}}
$$

has the same numerator (i.e., $A a^{2}+B a+C$ ) in the proof of Proposition 3(a).
(b) $\pi_{i}^{o}<\pi_{i}^{n}$ because $\pi_{i}^{o}-\pi_{i}^{n}=-\frac{\delta(\delta+\theta+2)\left[2(a+7) \delta^{2}+a \delta(\theta+4)+8 \delta(\theta+6)+8(\theta+4)\right](a(2 \delta+\theta+4)+2 \delta)}{4(a+2)(2 \delta+\theta+4)^{2}\left[3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right]^{2}}<0$ always holds.

Proof for Corollary 1: From Propositions 3(b) and 4(b), we can learn that $\pi_{i}^{o, S O}>\pi_{i}^{n, S O}$ and $\pi_{i}^{n, S T}>\pi_{i}^{o, S T}, i \in\{1,2\}$. We can also show that $\pi_{i}^{n, S O}>\pi_{i}^{n, S T}$ because $\pi_{i}^{n, S O}-\pi_{i}^{n, S T}=$ $\frac{(1-2 \alpha)((a+4) \delta+4)^{2}(\delta+\theta+2)}{4(a+2)\left[3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right]^{2}}>0$ always holds.

## A5 Proof of Proposition 5

Proof: We analyze the platform's incentive to introduce its own brand in two pricing contracts by comparing its profit across different scenarios.
(a) In the benchmark scenario, we can learn the platform's profit in sell-on and sell-to contract (i.e., $\pi_{r}^{s, S O}$ and $\pi_{r}^{s, S T}$ ) from Equations (19) and (21), respectively. In the own brand scenario, we can learn the platform's profit in sell-on and sell-to contract (i.e., $\pi_{r}^{o, S O}$ and $\pi_{r}^{o, S T}$ ) from Equations (20) and (22), respectively. Comparing the platform's profits in benchmark and own brand scenarios, we have the following results:
(1) $\pi_{r}^{o, S O}>\pi_{r}^{s, S O}$ when $a>a_{51}$. Note that

$$
\pi_{r}^{o, S O}-\pi_{r}^{s, S O}=\frac{A a^{2}+B a+C}{2(a+2)(\theta+2)^{2}\left[\delta^{2}(3-\alpha)+2 \delta(\theta+6)+2(\theta+4)\right]^{2}},
$$

where $A=(\delta+1)(\theta+2)^{2}(2 \delta+\theta+4)^{2}-\delta^{2}(\delta+2)(\theta+2)^{2} \alpha>0$;

$$
\begin{aligned}
& B=-2 \delta^{3}(\theta+2)^{2} \alpha^{2}+(-\theta-1) \alpha\left(\delta^{2}(-(\alpha-3))+2 \delta(\theta+6)+2(\theta+4)\right)^{2}+2(3 \delta(\delta+4)+8) \delta(\theta+2)^{2} \alpha+8(\delta+1) \delta \theta(\theta+2)^{2} \alpha+ \\
& 4(\delta+1) \delta(\theta+2)^{2}(2 \delta+\theta+4) ; \\
& C=-2 \delta^{4}(\theta+1)(\alpha-3)^{2} \alpha+4 \delta^{3}\left((\theta(\theta+10)+8) \alpha^{2}-2(\theta(\theta+13)+10) \alpha+(\theta+2)^{2}\right) \\
&+4 \delta^{2}\left(2\left(\theta^{3}-23 \theta-16\right) \alpha+(\theta(\theta+6)+4) \alpha^{2}+(\theta+2)^{2}\right)+16 \delta(\theta-2)(\theta(\theta+4)+2) \alpha+8 \theta^{2}(\theta+3) \alpha .
\end{aligned}
$$

We can verify that this quadratic function of $A a^{2}+B a+C$ in $a$ has two real solutions (i.e., $\left.B^{2}-4 A C>0\right)$. We denote these two solutions of $a$, which leads to $A a^{2}+B a+C=0$, as $a_{51 \_1}$ and $a_{51 \_2}$, where $a_{51 \_1}=\frac{-B-\sqrt{B^{2}-4 A C}}{2 A}$, and $a_{51 \_2}=\frac{-B+\sqrt{B^{2}-4 A C}}{2 A}$. The base level demand $a$ should be non-negative. We can easily show that $a_{51 \_1}<0$ always holds, and $a_{51 \_2}$ can be positive. Therefore, $A a^{2}+B a+C>0$ always holds when $a>\max \left\{a_{51 \_2}, 0\right\}:=a_{51}$. In other words, $\pi_{r}^{o, S O}>\pi_{r}^{s, S O}$
when $a>a_{51}$.
(2) $\pi_{r}^{o, S T}>\pi_{r}^{s, S T}$ when $a>a_{52}$. Note that

$$
\pi_{r}^{o, S T}-\pi_{r}^{s, S T}=\frac{A^{\prime} a^{2}+B^{\prime} a+C^{\prime}}{16(a+2)(3 \delta+2)(\theta+2)^{2}(2 \delta+\theta+4)^{2}}
$$

where $A^{\prime}=4(\delta+2)(\theta+2)^{2}(2 \delta+\theta+4)^{2}>0$;
$B^{\prime}=16 \delta(\theta+2)^{2}(2 \delta+\theta+4)^{2}-4(3 \delta+2)(\theta+1)^{2}(2 \delta+\theta+4)^{2} ;$
$C^{\prime}=16(\theta+2)^{2}\left(4 \delta^{3}+\delta^{2}(4 \theta+11)+\delta\left(\theta^{2}+6 \theta+8\right)+(\theta+2)^{2}\right)-8(3 \delta+2)(\theta+1)^{2}(2 \delta+\theta+4)^{2}$.
We can verify that this quadratic function of $A^{\prime} a^{2}+B^{\prime} a+C^{\prime}$ in $a$ has two real solutions (i.e., $\left.B^{\prime 2}-4 A^{\prime} C^{\prime}>0\right)$. We denote these two solutions of $a$, which leads to $A^{\prime} a^{2}+B^{\prime} a+C^{\prime}=0$. as $a_{52 \_1}$ and $a_{52 \_2}$, where $a_{52 \_1}=\frac{-B^{\prime}-\sqrt{B^{\prime 2}-4 A^{\prime} C^{\prime}}}{2 A^{\prime}}$, and $a_{52 \_2}=\frac{-B^{\prime}+\sqrt{B^{\prime 2}-4 A^{\prime} C^{\prime}}}{2 A^{\prime}}$. The base level demand $a$ should be non-negative. We can easily show that $a_{52 \_1}<0$ always holds, and $a_{52 \_2}$ can be positive. Therefore, $A^{\prime} a^{2}+B^{\prime} a+C^{\prime}>0$ always holds when $a>\max \left\{a_{52 \_2}, 0\right\}:=a_{52}$. In other words, $\pi_{r}^{o, S T}>\pi_{r}^{s, S T}$ when $a>a_{52}$.
(b) In the own brand scenario, we can learn the platform's profit in sell-on and sell-to contract (i.e., $\pi_{r}^{o, S O}$ and $\pi_{r}^{o, S T}$ ) from Equations (20) and (22), respectively. In the new-brand scenario, we can learn the platform's profit in sell-on and sell-to contract (i.e., $\pi_{r}^{n, S O}$ and $\pi_{r}^{n, S T}$ ) from Equations (23) and (24), respectively. Comparing the platform's profits in own brand and new-brand scenarios, we have the following results:
(1) $\pi_{r}^{o, S O}>\pi_{r}^{n, S O}$, because

$$
\pi_{r}^{o, S O}-\pi_{r}^{n, S O}=\frac{D a^{2}+E a+F}{(a+2)\left[3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right]^{2}\left[\delta^{2}(3-\alpha)+2 \delta(\theta+6)+2(\theta+4)\right]^{2}}>0 \text { always holds when } a \geq 0
$$

where

$$
\begin{aligned}
D= & -\delta^{2}(\delta+2) \alpha\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2}+(\delta+1)(2 \delta+\theta+4)^{2}\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2} \\
& +\alpha\left(-5 \delta^{3}-\delta^{2}(5 \theta+22)-\delta(\theta+4)(\theta+8)-(\theta+4)^{2}\right)\left(\delta^{2}(-(\alpha-3))+2 \delta(\theta+6)+2(\theta+4)\right)^{2}>0 \\
E= & -4(\delta+1) \delta^{5} \alpha^{3}(4 \delta+3 \theta+8)-2 \delta \alpha\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2}\left(5 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right) \\
& +4(\delta+1) \delta(2 \delta+\theta+4)\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2}+2 \delta^{3} \alpha^{2}\left(13 \delta^{2}+10(\delta+1) \theta+36 \delta+24\right)\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right) \\
F= & 4 \delta^{2}(\delta+1)\left(-\delta^{2} \alpha^{3}\left(5 \delta^{2}+4 \delta(\theta+3)+4(\theta+2)\right)+\alpha^{2}\left(7 \delta^{2}+6 \delta(\theta+2)+6 \theta+8\right)\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)\right. \\
& -\alpha\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2}+\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2} .
\end{aligned}
$$

Similarly, we can verify that this quadratic function of $D a^{2}+E a+F$ in $a$ has two real solutions (i.e., $E^{2}-4 D F>0$ ). There are two solutions of $a$ that lead to $D a^{2}+E a+F=0$. We verify that both solutions are less than 0 . Therefore, $\pi_{r}^{o, S O}>\pi_{r}^{s, S O}$ always holds.
(2) $\pi_{r}^{o, S T}>\pi_{r}^{n, S T}$, because

$$
\pi_{r}^{o, S T}-\pi_{r}^{n, S T}=\frac{D^{\prime} a^{2}+E^{\prime} a+F^{\prime}}{4(a+2)(3 \delta+2)\left[3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right]^{2}(2 \delta+\theta+4)^{2}}>0 \text { always holds }
$$

where

$$
\begin{aligned}
& D^{\prime}=(3 \delta+2)(2 \delta+\theta+4)^{2}\left(11 \delta^{3}+2 \delta^{2}(6 \theta+29)+3 \delta(\theta+4)(\theta+8)+3(\theta+4)^{2}\right)>0 \\
& E^{\prime}=4 \delta(3 \delta+2)(2 \delta+\theta+4)^{2}(\delta(7 \delta+5 \theta+24)+5 \theta+16)>0 \\
& F^{\prime}=4 \delta^{2}(3 \delta+2)\left(7(\delta+1) \theta^{2}+2 \delta(11 \delta+36) \theta+\delta(\delta(17 \delta+94)+160)+48 \theta+80\right)>0 .
\end{aligned}
$$

## A6 Proof of Proposition 6

Proof: Since the platform has the incentive to introduce its own brand when the base level demand is not very small under both pricing contracts, we then check when the platform prefers to introduce its own brand, what is its optimal pricing contract in the own brand scenario. Our analyses on the platform's optimal choice show the effect of own brand on independent sellers if the platform can choose whether to introduce own brand and use what contract.
(a) For the platform, $\pi_{r}^{o, S O}>\pi_{r}^{o, S T}$ if and only if $a<a_{6}$ and $\alpha>\alpha_{6}$. Note that

$$
\pi_{r}^{o, S T}-\pi_{r}^{o, S O}=\frac{A a^{2}+B a+C}{4(a+2)(3 \delta+2)(2 \delta+\theta+4)^{2}\left[\delta^{2}(3-\alpha)+2 \delta(\theta+6)+2(\theta+4)\right]^{2}}
$$

where

$$
\begin{aligned}
A= & (\delta+2)(2 \delta+\theta+4)^{2}\left(\delta^{2}(3-\alpha)+2 \delta(\theta+6)+2(\theta+4)\right)^{2}+4 \delta^{2}(\delta+2)(3 \delta+2) \alpha(2 \delta+\theta+4)^{2}-4(\delta+1)(3 \delta+2)(2 \delta+\theta+4)^{4}>0 \\
B= & 4 \delta(2 \delta+\theta+4)^{2}\left(\delta^{4}(\alpha-3)^{2}+\delta^{3}(-4 \theta(\alpha-3)+6(\alpha-7) \alpha+48)+4 \delta^{2}\left(\theta^{2}+\theta(12-7 \alpha)+(\alpha-25) \alpha+26\right)\right. \\
& +\delta\left(8 \theta^{2}-40 \theta \alpha+60 \theta-96 \alpha+96\right)+4(\theta+2)(\theta-4 \alpha+4) \\
C= & 16(3 \delta+2) \delta^{3} \alpha^{2}(2 \delta+\theta+4)^{2}+16(3 \delta+2) \delta^{2} \alpha^{2}(2 \delta+\theta+4)^{2}-16(\delta+1)(3 \delta+2) \delta^{2}(2 \delta+\theta+4)^{2}+4\left(4 \delta^{3}+\delta^{2}(4 \theta+11)+\delta\left(\theta^{2}+6 \theta+8\right)+(\theta+2)^{2}\right) \\
& \left(\delta^{2}(-(\alpha-3))+2 \delta(\theta+6)+2(\theta+4)\right)^{2}-64(3 \delta+2)(\delta(\delta+4)+5) \delta \alpha(2 \delta+\theta+4)^{2}-64(\delta+1)(3 \delta+2) \delta \theta \alpha(2 \delta+\theta+4)^{2} \\
& -128(3 \delta+2) \alpha(2 \delta+\theta+4)^{2}-64(\delta+1)(3 \delta+2) \theta \alpha(2 \delta+\theta+4)^{2} .
\end{aligned}
$$

We can verify that this quadratic function of $A a^{2}+B a+C$ in $a$ has two real solutions (i.e., $\left.B^{2}-4 A C>0\right)$. We denote these two solutions of $a$, which leads to $A a^{2}+B a+C=0$, as $a_{6 \_1}$ and $a_{6 \_2}$, where $a_{6 \_1}=\frac{-B+\sqrt{B^{2}-4 A C}}{2 A}, a_{6 \_2}=\frac{-B-\sqrt{B^{2}-4 A C}}{2 A}$. The base level demand $a$ should be non-negative. We can show that $a_{6_{-}} 2<0$ always holds, and $a_{6_{-}}>0$ if and only if $\alpha>\alpha_{6}$. Note that the condition $\alpha>\alpha_{6}$, which is a function of $\theta$ and $\delta$, ensures that $A a^{2}+B a+C<0$ when $a=0$. Therefore, $A a^{2}+B a+C<0$ if and only if $a<a_{6 \_1}:=a_{6}$ and $\alpha>\alpha_{6}$. In other words, $\pi_{r}^{o, S O}>\pi_{r}^{o, S T}$ if and only if $a<a_{6}$ and $\alpha>\alpha_{6}$.

We can also show that $\pi_{i}^{o, S O}>\pi_{i}^{o, S T}$ always holds. Note that

$$
\pi_{i}^{o, S O}-\pi_{i}^{o, S T}=\frac{D a^{2}+E a+F}{2(a+2)(2 \delta+\theta+4)^{2}\left(-\left(\delta^{2}(\alpha-3)\right)+2 \delta(\theta+6)+2(\theta+4)\right)^{2}},
$$

where
$D=\delta^{2}(1-\alpha)(2 \delta+\theta+4)^{2}>0$;
$E=8 \delta^{2}(1-\alpha)(2 \delta+\theta+4)^{2}+8 \delta(1-\alpha)(2 \delta+\theta+4)^{2}$
$F=16 \delta^{2}(1-\alpha)(2 \delta+\theta+4)^{2}-2\left(-\left(\delta^{2}(\alpha-3)\right)+2 \delta(\theta+6)+2(\theta+4)\right)^{2}+32 \delta(1-\alpha)(2 \delta+\theta+4)^{2}+16(1-\alpha)(2 \delta+\theta+4)^{2}$
We can verify that this quadratic function of $D a^{2}+E a+F$ in $a$ has two real solutions (i.e., $E^{2}-4 D F>0$ ), and those two solutions are all negative.
(b) Based on the consequences of Propositions 1, 2, 5, and the condition of part (a), we can show that incumbent sellers always hurt with the own brand induction in the sell-on and sell-to contracts.
(1) Proposition 1 shows that $\pi_{i}^{o, S O}>\pi_{i}^{s, S O}$ if and only if (i) $a<a_{11}$ and $\delta<\delta_{1}$ or (ii) $a>a_{12}$. However, when the platform prefers the own brand scenario with the sell-on contract, $a_{51}<a<a_{6}$ always holds, where $a_{51}$ and $a_{6}$ are defined in the proofs of Propositions 5 and 6(a), respectively. We can also verify that $a_{11}<a_{51}$ and $a_{6}<a_{12}$ always hold. Therefore, incumbent sellers always hurt with the own brand introduction in the sell-on contract (i.e., $\pi_{i}^{o, S O}<\pi_{i}^{s, S O}$ ) when the platform prefers the own brand scenario with the sell-on contract.
(2) Proposition 2 shows that $\pi_{i}^{o, S T}>\pi_{i}^{s, S T}$ if and only if $a<a_{2}$ and $\delta<\delta_{2}$. However, when the platform prefers the own brand scenario with the sell-to contract, $a>a_{52}$ and $a>a_{6}$ always hold, where $a_{52}$ and $a_{6}$ are defined in the proofs of Propositions 5 and 6(a), respectively. We can also verify that $a_{21}<a_{52}$ always holds. Therefore, incumbent sellers always hurt with the own brand introduction in the sell-to contract (i.e., $\pi_{i}^{o, S T}<\pi_{i}^{s, S T}$ ) when the platform prefers the own brand scenario with the sell-to contract.

## A7 Proof of Proposition 7

Proof: Under the sell-on enforcement, the platform makes its optimal choice based on the available cases: the benchmark and the new-brand scenarios under the sell-on contract (i.e., $\pi_{r}^{s, S O}$, and $\pi_{r}^{n, S O}$ ) and three scenarios under the sell-to contract (i.e., $\pi_{r}^{s, S T}, \pi_{r}^{o, S T}, \pi_{r}^{n, S T}$ ). As a consequence
of Proposition 5, when the platform prefers to introduce the own brand, the platform always has the incentive to introduce its own brand under the sell-to contract (i.e., $\pi_{r}^{o, S T}>\max \left\{\pi_{r}^{s, S T}, \pi_{r}^{n, S T}\right\}$ ). That is, under the sell-to contract, the platform would consider the own brand scenario only for the platform's profit comparison across contracts. Therefore, we compare its profit of the three cases (i.e., $\pi_{r}^{o, S T}, \pi_{r}^{n, S O}$, and $\pi_{r}^{s, S O}$ ).
(a) Under the sell-on enforcement, the platform prefers to sell in the benchmark scenario with a sell-on contract (i.e., $\pi_{r}^{s, S O}>\pi_{r}^{o, S T}$ and $\pi_{r}^{s, S O}>\pi_{r}^{n, S O}$ ) if and only if $a<a_{7}$ and $\alpha>\alpha_{7}$ because (i) $\pi_{r}^{s, S O}>\pi_{r}^{o, S T}$ if and only if $a<a_{7}$ and $\alpha>\alpha_{7}$; and (ii) $\pi_{r}^{s, S O}>\pi_{r}^{n, S O}$ always holds when $\pi_{r}^{n, S O}>\pi_{r}^{o, S T}$. Specifically,
(i) $\pi_{r}^{s, S O}>\pi_{r}^{o, S T}$ if and only if $a<a_{7}$ and $\alpha>\alpha_{7}$. Note that

$$
\pi_{r}^{o, S T}-\pi_{r}^{s, S O}=\frac{A a^{2}+B a+C}{4(a+2)(3 \delta+2)(\theta+2)^{2}(2 \delta+\theta+4)^{2}},
$$

where $A=(\delta+2)(\theta+2)^{2}(2 \delta+\theta+4)^{2}>0$;
$B=4 \delta(\theta+2)^{2}(2 \delta+\theta+4)^{2}-4(3 \delta+2)(\theta+1) \alpha(2 \delta+\theta+4)^{2}$;
$C=4(\theta+2)^{2}\left(4 \delta^{3}+\delta^{2}(4 \theta+11)+\delta\left(\theta^{2}+6 \theta+8\right)+(\theta+2)^{2}\right)-8(3 \delta+2)(\theta+1) \alpha(2 \delta+\theta+4)^{2}$.
We can verify that this quadratic function of $A a^{2}+B a+C$ in $a$ can has two real solutions. We denote these two solutions of $a$, which leads to $A a^{2}+B a+C=0$, as $a_{71 \_1}$ and $a_{71 \_2}$, where $a_{71 \_1}=\frac{-B+\sqrt{B^{2}-4 A C}}{2 A}$, and $a_{71 \_2}=\frac{-B-\sqrt{B^{2}-4 A C}}{2 A}$. The base level demand $a$ should be nonnegative. We can show that $a_{71 \_2}<0$, and $a_{71 \_1}>0$ if and only if $\alpha>\alpha_{7}$, where $\alpha>\alpha_{7}$ is a function of $\delta$ and $\theta$. Therefore, $A a^{2}+B a+C<0$ if and only if $a<a_{71 \_1}$ and $\alpha>\alpha_{7}$. In other words, $\pi_{r}^{s, S O}>\pi_{r}^{o, S T}$ if and only if $a<a_{71 \_} 1:=a_{7}$ and $\alpha>\alpha_{7}$.
(ii) $\pi_{r}^{s, S O}>\pi_{r}^{n, S O}$ always holds when $\pi_{r}^{n, S O}>\pi_{r}^{o, S T}$. Note that

$$
\pi_{r}^{o, S T}-\pi_{r}^{n, S O}=\frac{D a^{2}+E a+F}{4(a+2)(3 \delta+2)(2 \delta+\theta+4)^{2}\left[3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right]^{2}},
$$

where

$$
\begin{aligned}
D= & (2 \delta+\theta+4)^{2}\left[(\delta+2)\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2}-4(3 \delta+2) \alpha\left(5 \delta^{3}+\delta^{2}(5 \theta+22)+\delta(\theta+4)(\theta+8)+(\theta+4)^{2}\right)\right]>0 ; \\
E= & 4 \delta(2 \delta+\theta+4)^{2}\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2}-16 \delta(\delta+1)(3 \delta+2) \alpha(2 \delta+\theta+4)^{2}(4 \delta+3 \theta+8) ; \\
F= & 4\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2}\left(4 \delta^{3}+\delta^{2}(4 \theta+11)+\delta\left(\theta^{2}+6 \theta+8\right)+(\theta+2)^{2}\right) \\
& -16(\delta+1)(3 \delta+2) \alpha(2 \delta+\theta+4)^{2}\left(5 \delta^{2}+4 \delta(\theta+3)+4(\theta+2)\right) .
\end{aligned}
$$

We can verify that this quadratic function of $D a^{2}+E a+F$ in $a$ can have two real solutions (i.e.,
$E^{2}-4 D F>0$ ). We denote these two solutions of $a$, which leads to $D a^{2}+E a+F=0$, as $a_{72 \_1}$ and $a_{72 \_2}$, where $a_{72 \_1}=\frac{-E+\sqrt{E^{2}-4 D F}}{2 D}$, and $a_{72 \_2}=\frac{-E-\sqrt{E^{2}-4 D F}}{2 D}$. The base level demand $a$ should be non-negative. We can show that $a_{72 \_2}<0$, and $a_{72}{ }^{2} 1$ can be non-negative. Thus, $\pi_{r}^{n, S O}>\pi_{r}^{o, S T}$ if and only if $a<a_{72 \_1}$ and $a_{72 \_1}$ is non-negative. Following the similar steps we can show that

$$
\pi_{r}^{s, S O}-\pi_{r}^{n, S O}=\frac{G a^{2}+H a+I}{(\mathrm{a}+2)(\theta+2)^{2}\left[3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right]^{2}},
$$

where
$G=-\left((\theta+2)^{2}\left(\delta^{2}(5 \theta+22)+5 \delta^{3}+\delta(\theta+4)(\theta+8)+(\theta+4)^{2}\right)<0 ;\right.$
$H=(\theta+1)\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2}-4 \delta(\delta+1)(\theta+2)^{2}(4 \delta+3 \theta+8) ;$
$I=2(\theta+1)\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2}-4(\delta+1)(\theta+2)^{2}\left(5 \delta^{2}+4 \delta(\theta+3)+4(\theta+2)\right)$
We can verify that this quadratic function of $G a^{2}+H a+I$ in $a$ can have two real solutions (i.e., $\left.H^{2}-4 G I>0\right)$. We denote these two solutions of $a$, which leads to $G a^{2}+H a+I=0$, as $a_{73 \_1}$ and $a_{73 \_2}$, where $a_{73 \_2}=\frac{-H+\sqrt{H^{2}-4 G I}}{2 G}$, and $a_{73} \__{1}=\frac{-H-\sqrt{H^{2}-4 G I}}{2 G}$. The base level demand $a$ should be non-negative. We can show that $a_{73 \_1}>a_{73 \_2}$, where $a_{73 \_1}$ is always positive and $a_{73 \_2}$ can be non-negative. In other words, $\pi_{r}^{n, S O}>\pi_{r}^{s, S O}$ if and only if $a<a_{73 \_2}$ or $a>a_{73 \_1}$. Moreover, we can verify that $a_{73_{-} 1}>a_{72 \_1}$ always holds. Thus, $\pi_{r}^{n, S O}>\pi_{r}^{s, S O}$ and $\pi_{r}^{n, S O}>\pi_{r}^{o, S T}$ if and only if $a<\min \left(a_{72 \_1}, a_{73 \_2}\right)$, and $a_{72 \_1}$ and $a_{73 \_2}$ are non-negative. However, such market condition will never be met when platform prefers to introduce own brand product because we can verify that $a_{73 \_2}<\max \left(a_{51}, a_{52}\right)$ always holds. Therefore, under the sell-on enforcement, the platform prefers to stay under the sell-on contract only with two incumbents if and only if $a<a_{7}$ and $\alpha>\alpha_{7}$.
(b) When the conditions in part (a) are not satisfied, the platform has the incentive to introduce introduce own brand under the sell-to contract (i.e., $\pi_{r}^{o, S T}>\pi_{r}^{s, S O}$ and $\pi_{r}^{o, S T}>\pi_{r}^{n, S O}$ ) when $a>a_{7}$. Note that while Propositions 2 describes that the incumbent sellers can benefit from the introduction of own brand product when $a<a_{2}$ and $\delta<\delta_{2}$, such conditions cannot be met when the platform prefers to introduce own brand with the sell-to contract because $a_{2}<a_{7}$ always holds. Therefore, the platform's best response (i.e., keeping the sell-to contract with its own brand introduction) always hurts the incumbent sellers.

## A8 Proof of Proposition 8

Proof: Under the sell-to enforcement, the platform makes its optimal choice based on the available cases: the benchmark and the new-brand scenarios under the sell-to contract (i.e., $\pi_{r}^{s, S T}$ and $\pi_{r}^{n, S T}$ ), and three scenarios under the sell-on contract (i.e., $\pi_{r}^{s, S O}, \pi_{r}^{n, S O}, \pi_{r}^{o, S O}$ ). As a consequence of Proposition 5, when the platform prefers to introduce the own brand, the platform's profit in the own brand scenario is always higher than in the other two scenarios (i.e., $\pi_{r}^{o, S O}>\max \left\{\pi_{r}^{s, S O}\right.$, $\left.\left.\pi_{r}^{n, S O}\right\}\right)$. That is, under the sell-on contract, the platform would consider the own brand scenario only for the platform's profit comparison across contracts. Therefore, we compare the platform's profit of the three cases (i.e., $\pi_{r}^{s, S T}, \pi_{r}^{n, S T}$, and $\pi_{r}^{o, S O}$ ).
(a) Under the sell-to enforcement, the platform prefers to sell in the benchmark scenario with a sell-to contract (i.e., $\pi_{r}^{s, S T}>\pi_{r}^{o, S O}$ and $\pi_{r}^{s, S T}>\pi_{r}^{n, S T}$ ) if and only if $a<\min \left(a_{81}, a_{82}\right), \theta>\theta_{8}$, and $\alpha<\alpha_{8}$ because (i) $\pi_{r}^{s, S T}>\pi_{r}^{o, S O}$ if and only if $a<a_{81}$ and $\alpha<\alpha_{8}$; and (ii) $\pi_{r}^{s, S T}>\pi_{r}^{n, S T}$ if and only if $a<a_{82}$ and $\theta>\theta_{8}$. Specifically,
(i) $\pi_{r}^{s, S T}>\pi_{r}^{o, S O}$ if and only if $a<a_{81}$ and $\alpha<\alpha_{8}$. Note that

$$
\pi_{r}^{o, S O}-\pi_{r}^{s, S T}=\frac{A a^{2}+B a+C}{4(a+2)(\theta+2)^{2}\left[\delta^{2}(3-\alpha)+2 \delta(\theta+6)+2(\theta+4)\right]^{2}},
$$

where $A=4(\delta+1)(\theta+2)^{2}(2 \delta+\theta+4)^{2}-4 \delta^{2}(\delta+2)(\theta+2)^{2} \alpha>0$;
$B=-8 \delta^{3}(\theta+2)^{2} \alpha^{2}-(\theta+1)^{2}\left(\delta^{2}(-(\alpha-3))+2 \delta(\theta+6)+2(\theta+4)\right)^{2}+8(3 \delta(\delta+4)+8) \delta(\theta+2)^{2} \alpha+32(\delta+1) \delta \theta(\theta+2)^{2} \alpha+$ $16(\delta+1) \delta(\theta+2)^{2}(2 \delta+\theta+4) ;$
$C=-16 \delta^{3}(\theta+2)^{2} \alpha^{2}-16 \delta^{2}(\theta+2)^{2} \alpha^{2}-2(\theta+1)^{2}\left(\delta^{2}(-(\alpha-3))+2 \delta(\theta+6)+2(\theta+4)\right)^{2}+16(\delta+1) \delta^{2}(\theta+2)^{2}+64(\delta(\delta+4)+$ 5) $\delta(\theta+2)^{2} \alpha+64(\delta+1) \delta \theta(\theta+2)^{2} \alpha+64(\delta+1) \theta(\theta+2)^{2} \alpha+128(\theta+2)^{2} \alpha$.

We can verify that this quadratic function of $A a^{2}+B a+C$ in $a$ has two real solutions (i.e., $B^{2}-4 A C>0$ ). We denote these two solutions of $a$, which leads to $A a^{2}+B a+C=0$, as $a_{81 \_1}$ and $a_{81 \_2}$, where $a_{81 \_1}=\frac{-B+\sqrt{B^{2}-4 A C}}{2 A}$, and $a_{81 \_2}=\frac{-B-\sqrt{B^{2}-4 A C}}{2 A}$. The base level demand $a$ should be non-negative. We can show that $a_{81 \_2}<0$, and $a_{81 \_1}>0$ if and only if $\alpha<\min \left(\alpha_{81}, \frac{1}{2}\right)$, where $\alpha_{81}$ is a function of $\delta$ and $\theta$. Therefore, $A a^{2}+B a+C<0$ if and only if $a<a_{81}{ }^{1}$ and $\alpha<\min \left(\alpha_{81}, \frac{1}{2}\right)$. In other words, $\pi_{r}^{s, S T}>\pi_{r}^{o, S O}$ if and only if $a<a_{81 \_1}:=a_{81}$ and $\alpha<\min \left(\alpha_{81}, \frac{1}{2}\right):=\alpha_{8}$.
(ii) $\pi_{r}^{s, S T}>\pi_{r}^{n, S T}$ if and only if $a<a_{82}$ and $\theta>\theta_{8}$. Note that

$$
\pi_{r}^{n, S T}-\pi_{r}^{s, S T}=\frac{D a^{2}+E a+F}{16(a+2)(3 \delta+2)(\theta+2)^{2}\left[3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right]^{2}},
$$

where $D=(\delta+1)(\theta+2)^{2}\left((\delta(4 \delta+3)+2) \theta^{2}+4(3 \delta(\delta+1)+2)(\delta+2) \theta+(3 \delta(3 \delta+4)+8)(\delta+2)^{2}\right)>0$;
$E=16 \delta(\delta+1)(\theta+2)^{2}\left(4(\delta+1) \theta^{2}+(3 \delta(4 \delta+11)+22) \theta+(\delta+2)(3 \delta+4)^{2}\right)-4(3 \delta+2)(\theta+1)^{2}\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2} ;$
$F=16(\delta+1)^{2}(\theta+2)^{2}\left(9 \delta^{3}+6 \delta^{2}(2 \theta+5)+4 \delta(\theta+2)(\theta+4)+4(\theta+2)^{2}\right)-8(3 \delta+2)(\theta+1)^{2}\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2}$.
We can verify that this quadratic function of $D a^{2}+E a+F$ in $a$ has two real solutions (i.e., $\left.E^{2}-4 D F>0\right)$. We denote these two solutions of $a$, which leads to $D a^{2}+E a+F=0$, as $a_{82 \_1}$ and $a_{81 \_2}$, where $a_{81 \_1}=\frac{-E+\sqrt{E^{2}-4 D F}}{2 D}$ and $a_{81 \_2}=\frac{-E-\sqrt{E^{2}-4 D F}}{2 D}$. The base level demand $a$ should be non-negative. We can show that $a_{81 \_2}<0$, and $a_{81 \_1}>0$ if and only if $\theta>\theta_{8}$. Therefore, $D a^{2}+E a+F<0$ if and only if $a<a_{82_{-} 1}$ and $\theta>\theta_{8}$. In other words, $\pi_{r}^{s, S T}>\pi_{r}^{n, S T}$ if and only if $a<a_{82 \_1}:=a_{82}$ and $\theta>\theta_{8}$.

Altogether, $\pi_{r}^{s, S T}>\max \left\{\pi_{r}^{o, S O}, \pi_{r}^{n, S T}\right\}$ if and only if $a<\min \left(a_{81}, a_{82}\right), \theta>\theta_{8}$, and $\alpha<\alpha_{8}$.
(b) When the conditions in part (a) are not satisfied, the platform has the incentive to introduce new brand under the sell-to contract or introduce own brand under the sell-on contract (i.e., $\left.\min \left\{\pi_{r}^{n, S T}, \pi_{r}^{o, S O}\right\}>\pi_{r}^{s, S T}\right)$. First, we consider the case when the platform prefers a new brand under the sell-to contract (i.e., $\pi_{r}^{n, S T}>\pi_{r}^{o, S O}$ ). Within this condition, independent sellers always hurt because of the new brand (i.e., $\pi_{i}^{n, S T}<\pi_{i}^{s, S T}$ ). Note that while Propositions 3 and 4 (both having the same condition) describe that incumbent sellers can benefit from the new brand introduction under certain market conditions, such conditions cannot be met when the platform prefers new brand. Therefore, we only consider the other case when the platform prefers own brand under the sell-on contract (i.e., $\pi_{r}^{n, S T}<\pi_{r}^{o, S O}$ ). With the own brand under the sell-on contract, independent sellers can benefit from the own brand (i.e., $\pi_{i}^{o, S O}>\pi_{i}^{s, S O}$ ) if and only if (i) $a<a_{11}$ and $\delta<\delta_{1}$ or (ii) $a>a_{12}$, as described in Proposition 1. However, the first condition (i.e., $a<a_{11}$ and $\delta<\delta_{1}$ ) cannot be met when the platform prefers to introduce own brand with the sell-on contract because $a_{11}<a_{81}$ always holds. Therefore, under the condition (ii), the policy can be helpful for the incumbents, since $a_{81}<a_{12}$ always holds.

## A9 Proof of Proposition 9

Proof: Under the enforcement against the platform's brand under both contracts, the platform makes its optimal choice based on the available four cases: the benchmark and new-brand scenarios under both contracts (i.e., $\pi_{r}^{s, S T}$ and $\pi_{r}^{n, S T}$ for sell-to; and $\pi_{r}^{s, S O}$ and $\pi_{r}^{n, S O}$ for sell-on). Different
from the proofs of Propositions 7 and 8, we compare the platform's profit in a pairwise manner among these four cases.
(a) The platform prefers to keep the two incumbents (i.e., benchmark scenario $s$ ) under the sell-on contract (i.e., $\pi_{r}^{s, S O}>\pi_{r}^{s, S T}, \pi_{r}^{s, S O}>\pi_{r}^{n, S T}$, and $\pi_{r}^{s, S O}>\pi_{r}^{n, S O}$ ) if and only if $a<\min \left(a_{91}, a_{92}\right)$, $\theta<\delta$, and $\alpha>\frac{\theta+1}{4}$ because (i) $\pi_{r}^{s, S O}>\pi_{r}^{s, S T}$ if and only if $\alpha>\frac{\theta+1}{4}$; (ii) $\pi_{r}^{s, S O}>\pi_{r}^{n, S O}$ if and only if $a<a_{73_{-} 1}:=a_{91}$; and (iii) $\pi_{r}^{s, S O}>\pi_{r}^{n, S T}$ if and only if $a<a_{92}$ and $\alpha>\alpha_{91}$. Note that $\tilde{\alpha}$ is function of $\theta$ and $\delta$. Moreover, we can show that $\alpha_{91}<\frac{\theta+1}{4}$ always holds. Specifically,
(i) $\pi_{r}^{s, S O}>\pi_{r}^{s, S T}$ if and only if $\pi_{r}^{s, S O}-\pi_{r}^{s, S T}=-\frac{(\theta+1)(\theta-4 \alpha+1)}{4(\theta+2)^{2}}>0$, which leads to $\alpha>\frac{\theta+1}{4}$.
(ii) When platform prefers to choose the scenario $o, \pi_{r}^{s, S O}>\pi_{r}^{n, S O}$ if and only if $a<a_{73} \_1:=a_{91}$, which is shown through the proof of Proposition 7(a)-ii.
(iii) $\pi_{r}^{s, S O}>\pi_{r}^{n, S T}$ if and only if if $a<a_{92}$ and $\alpha>\alpha_{91}$. Note that

$$
\pi_{r}^{n, S T}-\pi_{r}^{s, S O}=\frac{D a^{2}+E a+F}{4(a+2)(3 \delta+2)(\theta+2)^{2}\left[3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right]^{2}},
$$

where $D=(\delta+1)(\theta+2)^{2}\left((\delta(4 \delta+3)+2) \theta^{2}+4(3 \delta(\delta+1)+2)(\delta+2) \theta+(3 \delta(3 \delta+4)+8)(\delta+2)^{2}\right)>0$;
$E=4 \delta(\delta+1)(\theta+2)^{2}\left(4(\delta+1) \theta^{2}+(3 \delta(4 \delta+11)+22) \theta+(\delta+2)(3 \delta+4)^{2}\right)-4(3 \delta+2)(\theta+1) \alpha\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2} ;$ $F=4(\delta+1)^{2}(\theta+2)^{2}\left(9 \delta^{3}+6 \delta^{2}(2 \theta+5)+4 \delta(\theta+2)(\theta+4)+4(\theta+2)^{2}\right)-8(3 \delta+2)(\theta+1) \alpha\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2}$.

We can verify that this quadratic function of $D a^{2}+E a+F$ in $a$ has two real solutions (i.e., $E^{2}-4 D F>0$ ). We denote these two solutions of $a$, which leads to $D a^{2}+E a+F=0$, as $a_{92}{ }^{2} 1$ and $a_{92 \_2}$, where $a_{92 \_1}=\frac{-E+\sqrt{E^{2}-4 D F}}{2 D}$, and $a_{92 \_2}=\frac{-E-\sqrt{E^{2}-4 D F}}{2 D}$. The base level demand $a$ should be non-negative. We can show that $a_{92 \_2}<0$, and $a_{92 \_1}>0$ if and only if $\alpha>\alpha_{91}$, where $\alpha_{91}$ is a function of $\theta$ and $\delta$. In other words, $\pi_{r}^{s, S O}>\pi_{r}^{n, S T}$ if and only if $a<a_{92}{ }^{1}:=a_{92}$ and $\alpha>\alpha_{91}$. But, we can rule out the condition of $\alpha>\alpha_{91}$, because of $\alpha_{91}<\frac{\theta+1}{4}$.
(b) The platform prefers to keep the two incumbents (i.e., benchmark scenario $s$ ) under the sellto contract (i.e., $\pi_{r}^{s, S T}>\pi_{r}^{s, S O}, \pi_{r}^{s, S T}>\pi_{r}^{n, S T}$, and $\pi_{r}^{s, S T}>\pi_{r}^{n, S O}$ ) if and only if $a<\min \left(a_{93}, a_{82}\right)$, $\theta>\theta_{8}$, and $\alpha<\frac{\theta+1}{4}$ because (i) $\pi_{r}^{s, S T}>\pi_{r}^{s, S O}$ if and only if $\alpha<\frac{\theta+1}{4}$; (ii) $\pi_{r}^{s, S T}>\pi_{r}^{n, S T}$ if and only if $a<a_{82}$ and $\theta>\theta_{8}$; and (iii) $\pi_{r}^{s, S T}>\pi_{r}^{n, S O}$ if and only if $a<a_{93}$ and $\alpha<\alpha_{92}$. Moreover, we can show that $\alpha_{92}>\frac{\theta+1}{4}$ always holds. Specifically,
(i) $\pi_{r}^{s, S T}>\pi_{r}^{s, S O}$ if and only if $\pi_{r}^{s, S T}-\pi_{r}^{s, S O}=\frac{(1+\theta)^{2}}{4(2+\theta)^{2}}-\frac{\alpha(1+\theta)^{2}}{(2+\theta)^{2}}>0$, which leads to $\alpha<\frac{\theta+1}{4}$.
(ii) $\pi_{r}^{s, S T}>\pi_{r}^{n, S T}$ if and only if $a<a_{82}$ and $\theta>\theta_{8}$, which is shown through the analysis in Proposition 8(a).
(iii) $\pi_{r}^{s, S T}>\pi_{r}^{n, S O}$ if and only if $a<a_{93}$ and $\alpha<\alpha_{92}$. Note that

$$
\pi_{r}^{n, S O}-\pi_{r}^{s, S T}=\frac{G a^{2}+H a+I}{4(a+2)(\theta+2)^{2}\left[3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right]^{2}},
$$

where $G=4(\theta+2)^{2} \alpha\left(5 \delta^{3}+\delta^{2}(5 \theta+22)+\delta(\theta+4)(\theta+8)+(\theta+4)^{2}\right)>0$;
$H=16 \delta(\delta+1)(\theta+2)^{2} \alpha(4 \delta+3 \theta+8)-(\theta+1)^{2}\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2} ;$
$I=16(\delta+1)(\theta+2)^{2} \alpha\left(5 \delta^{2}+4 \delta(\theta+3)+4(\theta+2)\right)-2(\theta+1)^{2}\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2}$.
We can verify that this quadratic function of $G a^{2}+H a+I$ in $a$ has two real solutions (i.e., $\left.H^{2}-4 G I>0\right)$. We denote these two solutions of $a$, which leads to $G a^{2}+H a+I=0$, as $a_{93}{ }^{1} 1$ and $a_{93 \_2}$, where $a_{93}{ }^{1}=\frac{-H+\sqrt{H^{2}-4 G I}}{2 G}$, and $a_{93 \_2}=\frac{-H-\sqrt{H^{2}-4 G I}}{2 G}$. The base level demand $a$ should be non-negative. We can show that $a_{93 \_2}<0$, and $a_{93}{ }_{-1}>0$ if and only if $\alpha<\alpha_{92}$. Therefore, $G a^{2}+H a+I<0$ if and only if $a<a_{93^{2}} 1$ and $\alpha<\alpha_{92}$. In other words, $\pi_{r}^{s, S T}>\pi_{r}^{n, S O}$ if and only if $a<a_{93}{ }^{1} 1:=a_{93}$ and $\alpha<\alpha_{92}$. But, we can rule out the condition of $\alpha>\alpha_{92}$, because of $\alpha_{92}>\frac{\theta+1}{4}$.

The proofs of (c) and (d) are as follows:
When the conditions in parts (a) and (b) are not satisfied, the platform has the incentive to introduce new brand under the sell-to contract or under the sell-on contract. We can show that $\pi_{r}^{n, S O}>\pi_{r}^{n, S T}$ if and only if $\alpha>\alpha_{9}$. Note that

$$
\pi_{r}^{n, S O}-\pi_{r}^{n, S T}=\frac{\left(A^{\prime} a^{2}+B^{\prime} a+C^{\prime}\right)(3 \delta+2) \alpha-\left(D^{\prime} a^{2}+E^{\prime} a+F^{\prime}\right)(1+\delta)}{4(a+2)(3 \delta+2)\left[3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right]^{2}},
$$

where $A^{\prime}=5 \delta^{3}+\delta^{2}(5 \theta+22)+\delta(\theta+4)(\theta+8)+(\theta+4)^{2}>0$;
$B^{\prime}=4 \delta(\delta+1)(4 \delta+3 \theta+8)>0$;
$C^{\prime}=4(\delta+1)\left(5 \delta^{2}+4 \delta(\theta+3)+4(\theta+2)\right)>0 ;$
$D^{\prime}=(\delta(4 \delta+3)+2) \theta^{2}+4(3 \delta(\delta+1)+2)(\delta+2) \theta+(3 \delta(3 \delta+4)+8)(\delta+2)^{2}>0 ;$
$E^{\prime}=4 \delta\left(4(\delta+1) \theta^{2}+(3 \delta(4 \delta+11)+22) \theta+(\delta+2)(3 \delta+4)^{2}\right)>0 ;$
$F^{\prime}=4(\delta+1)\left(9 \delta^{3}+6 \delta^{2}(2 \theta+5)+4 \delta(\theta+2)(\theta+4)+4(\theta+2)^{2}\right)>0$.
Therefore, $\pi_{r}^{n, S O}>\pi_{r}^{n, S T}$ if and only if $\alpha>\frac{\left(D^{\prime} a^{2}+E^{\prime} a+F^{\prime}\right)(1+\delta)}{\left(A^{\prime} a^{2}+B^{\prime} a+C^{\prime}\right)(3 \delta+2)}:=\alpha_{9}$. Moreover, we can show that $\frac{\left(D^{\prime} a^{2}+E^{\prime} a+F^{\prime}\right)(1+\delta)}{\left(A^{\prime} a^{2}+B^{\prime} a+C^{\prime}\right)(3 \delta+2)}<\frac{1}{2}$ always holds.

Combining with the results from Propositions 3(a) and 4(a), our results indicate that when the
platform's optimal case is to introduce new brand under the sell-to contract or under the sell-on contract after the own brand is prohibited in both contracts, incumbent sellers benefits from the introduction of new brand if and only if (i) $a<a_{31}$ and $\delta<\delta_{3}$ or (ii) $a>a_{32}$. However, the first condition (i.e., $a<a_{31}$ and $\delta<\delta_{3}$ ) cannot be met when the platform prefers to introduce own brand because $a_{31}<\max \left(a_{51}, a_{52}\right)$ always holds.

## A10 Platform's Decision of Commission Rates

In this extension, we check our results by considering the case when the commission rate is different across the scenarios (e.g., scenarios $o$ vs. $s$ ). We first confirm that our results are qualitatively the same if the commission rate is exogenous but it is significantly lower in scenario $s$ than in scenario $o$ (i.e., $0<\alpha^{s}<\alpha^{o}$ ).

Proof: Suppose that the commission rates in scenarios $o$ and $s$ are $\alpha^{o}$ and $\alpha^{s}$, respectively, where $0<\alpha^{s}<\alpha^{o}$. We can rewrite sellers' profit $\pi_{i}^{s}$ in Equation (19) and $\pi_{i}^{o}$ in Equation (20) as $\pi_{i}^{s}=\frac{(1+\theta)\left(1-\alpha^{s}\right)}{2(2+\theta)^{2}}$ and $\pi_{i}^{o}=\frac{\left(1-\alpha^{o}\right)((4+a) \delta+4)^{2}(2+\delta+\theta)}{2(2+a)\left(\delta^{2}\left(3-\alpha^{o}\right)+2 \delta(6+\theta)+2(4+\theta)\right)^{2}}$, respectively, $i \in\{1,2\}$. Note that

$$
\begin{aligned}
\pi_{i}^{o}-\pi_{i}^{s} & =\frac{\left(1-\alpha^{o}\right)((4+a) \delta+4)^{2}(2+\delta+\theta)}{2(2+a)\left(\delta^{2}\left(3-\alpha^{o}\right)+2 \delta(6+\theta)+2(4+\theta)\right)^{2}}-\frac{(1+\theta)\left(1-\alpha^{s}\right)}{2(2+\theta)^{2}} \\
& >\frac{\left(1-\alpha^{o}\right)((4+a) \delta+4)^{2}(2+\delta+\theta)}{2(2+a)\left(\delta^{2}\left(3-\alpha^{o}\right)+2 \delta(6+\theta)+2(4+\theta)\right)^{2}}-\frac{(1+\theta)}{2(2+\theta)^{2}}, \text { where } \alpha^{s}=0 \\
& =\frac{\left(A a^{2}+B a+C\right)}{2(a+2)(\theta+2)^{2}\left[\delta^{2}\left(3-\alpha^{o}\right)+2 \delta(\theta+6)+2(\theta+4)\right]^{2}},
\end{aligned}
$$

where $A=2 \delta^{2}(\theta+2)^{2}\left(1-\alpha^{o}\right)(\delta+\theta+2)>0$;
$B=16 \delta^{2}(\theta+2)^{2}\left(1-\alpha^{o}\right)(\delta+\theta+2)-2(\theta+1)\left(\delta^{2}\left(-\left(\alpha^{o}-3\right)\right)+2 \delta(\theta+6)+2(\theta+4)\right)^{2}+16 \delta(\theta+2)^{2}\left(1-\alpha^{o}\right)(\delta+\theta+2)$;
$\left.C=32 \delta^{2}(\theta+2)^{2}\left(1-\alpha^{o}\right)(\delta+\theta+2)+4(\theta+1)\left(\delta^{2}\left(-\alpha^{o}-3\right)\right)+2 \delta(\theta+6)+2(\theta+4)\right)^{2}+64 \delta(\theta+2)^{2}\left(1-\alpha^{o}\right)(\delta+\theta+2)+32(\theta+$ $2^{2}\left(1-\alpha^{o}\right)(\delta+\theta+2)$.

We can verify that this quadratic function of $A a^{2}+B a+C$ in $a$ has two real solutions (i.e., $\left.B^{2}-4 A C>0\right)$. We denote these two solutions of $a$, which leads to $A a^{2}+B a+C=0$, as $\underline{a}_{1}^{o}$ and $\underline{a}_{2}^{o}$, where $\underline{a}_{1}^{o}=\frac{-B+\sqrt{B^{2}-4 A C}}{2 A}$, and $\underline{a}_{2}^{o}=\frac{-B-\sqrt{B^{2}-4 A C}}{2 A}$. The base level demand $a$ should be non-negative. We can easily show that $\underline{a}_{1}^{o}>0$, and $\underline{a}_{2}^{o}>0$ if only if $\delta<\underline{\delta}^{o}$, where $\underline{\delta}^{o}$ is the only solution that is in the range of $(0,1)$ for $-B-\sqrt{B^{2}-4 A C}=0$. Therefore, $A a^{2}+B a+C>0$ if and only if (i) $a<\underline{a}_{2}^{o}$ and $\delta<\underline{\delta}^{o}$ or (ii) $a>\underline{a}_{1}^{o}$. In other words, $\frac{A a^{2}+B a+C}{2(a+2)(\theta+2)^{2}\left[\delta^{2}\left(3-\alpha^{o}\right)+2 \delta(\theta+6)+2(\theta+4)\right]^{2}}>0$ if and only if (i) $a<\underline{a}_{2}^{o}$ and $\delta<\underline{\delta}^{o}$ or (ii) $a>\underline{a}_{1}^{o}$.

Combining the result from the case where $\alpha^{s}=\alpha^{o}$ (i.e., Proposition 1), we can know that for
any $\alpha^{s}$ between 0 and $\alpha^{o}$, there is always a threshold value $\underline{a}^{o}$ such that
$\pi_{i}^{o}-\pi_{i}^{s}=\frac{\left(1-\alpha^{o}\right)((4+a) \delta+4)^{2}(2+\delta+\theta)}{2(2+a)\left[\delta^{2}\left(3-\alpha^{o}\right)+2 \delta(6+\theta)+2(4+\theta)\right]^{2}}-\frac{(1+\theta)\left(1-\alpha^{s}\right)}{2(2+\theta)^{2}}>0$ if and only if $a>\underline{a}^{o}, i \in\{1,2\}$.
We can also consider an intervention to protect the sellers at the least while satisfying the platform's incentive. In other words, the commission rate is set such that the sellers are indifferent between contracts, and the platform prefers the sell-on to sell-to contract. We examine this case by considering the commission rate at equilibrium is determined by the following rules: (1) with such a commission rate, the platform prefers the sell-on contract, and (2) with such a commission rate, independent sellers are indifferent between sell-on contract and sell-to contract. By following these rules, we can decide the commission rate in the benchmark scenario and own brand scenario.

In the benchmark scenario, the platform prefers the sell-on contract only if $\alpha>\frac{\theta+1}{4}$ and the independent sellers prefer to the sell-on contract as long as $\alpha<\frac{1}{2}$. Given that $\theta$ is between 0 and 1 , the e-commerce platform would set $\alpha=\frac{1}{2}$ because with such a commission rate, the platform would prefer the sell-on contract (rule (1)) and the independent seller is indifferent between the two pricing contracts (rule (2)). In the own brand scenario, we can learn that the platform prefers the sell-on contract only if $\alpha>\alpha_{6}$ from Proposition 6. Our analyses also show that the independent sellers will also prefer to sell under the sell-on contract when $\alpha<\underline{\alpha}^{\bullet}$, which is higher than $\alpha_{6}$ and $\frac{1}{2}$. In other words, the platform would set $\alpha=\underline{\alpha}^{\bullet}$ or $\frac{1}{2}$ because of rules (1) and (2) with such a commission rate. From our discussions at the main analyses, where the commission rate is the same in both scenario $o$ and $s$ (i.e., $\alpha^{o}=\alpha^{s}$ ), we can know the result is consistent with our main analysis when $\alpha=\frac{1}{2}$ in both scenarios. From our above discussions on the case where commission rate in scenario $s$ is lower than that in scenario $o$ (i.e., $\alpha^{o}>\alpha^{s}$ ), we can know that the result is consistent with our main analysis when $\alpha=\frac{1}{2}$ in scenario $s$ and $\alpha=\underline{\alpha}^{\bullet}$ in scenario $o$.

Proof: (a) In the benchmark scenario,
(i) $\pi_{r}^{s, S O}>\pi_{r}^{s, S T}$ if and only if $\pi_{r}^{s, S O}-\pi_{r}^{s, S T}=-\frac{(\theta+1)(\theta-4 \alpha+1)}{4(\theta+2)^{2}}>0$, which leads to $\alpha>\frac{\theta+1}{4}$;
(ii) $\pi_{i}^{s, S O}>\pi_{i}^{s, S T}, i \in\{1,2\}$ if and only if $\pi_{i}^{s, S O}-\pi_{i}^{s, S T}=\frac{(\theta+1)(1-2 \alpha)}{4(\theta+2)^{2}}>0$, which leads to $\alpha<\frac{1}{2}$.
(b) In the own brand scenario,
(i) From Proposition 6, we know that $\pi_{r}^{o, S O}>\pi_{r}^{o, S T}$ if and only if $a<a_{6}$ and $\alpha>\alpha_{6}$;
(ii) $\pi_{i}^{o, S O}>\pi_{i}^{o, S T}$ when $\alpha<\underline{\alpha}^{\bullet}, i \in\{1,2\}$. Note that

$$
\pi_{i}^{o, S O}-\pi_{i}^{o, S T}=\frac{(2+\delta+\theta)\left(A a^{2}+B a+C\right)}{2(a+2)(2 \delta+\theta+4)^{2}\left[\delta^{2}(3-\alpha)+2 \delta(\theta+6)+2(\theta+4)\right]^{2}}
$$

where $A=\delta^{2}(1-\alpha)(2 \delta+\theta+4)^{2}>0$;
$B=8 \delta(\delta+1)(2 \delta+\theta+4)^{2}(1-\alpha)$;
$C=16 \delta^{2}(1-\alpha)(2 \delta+\theta+4)^{2}-2\left(\delta^{2}(-(\alpha-3))+2 \delta(\theta+6)+2(\theta+4)\right)^{2}+32 \delta(1-\alpha)(2 \delta+\theta+4)^{2}+16(1-\alpha)(2 \delta+\theta+4)^{2}$.
We can verify that this quadratic function of $A a^{2}+B a+C$ in $a$ has two real solutions (i.e., $\left.B^{2}-4 A C>0\right)$. We denote these two solutions of $a$, which leads to $A a^{2}+B a+C=0$, as $a_{1}$ and $a_{2}$, where $a_{1}=\frac{-B+\sqrt{B^{2}-4 A C}}{2 A}$, and $a_{2}=\frac{-B-\sqrt{B^{2}-4 A C}}{2 A}$. The base level demand $a$ should be nonnegative. We can easily show that $a_{2}<0$, and $a_{1}>0$ if and only if $\alpha>\underline{\alpha}^{\bullet}$, where $\underline{\alpha}^{\bullet}$ is a function of $\delta$ and $\theta$, and we can show that $\underline{\alpha}^{\bullet}>\frac{1}{2}$. The condition $\alpha<\underline{\alpha}^{\bullet}$ ensures $A a^{2}+B a+C>0$ when $a=0$. Therefore, when $\alpha<\underline{\alpha}^{\bullet}, \pi_{i}^{o, S O}>\pi_{i}^{o, S T}$ because both two solutions of $A a^{2}+B a+C=0$ are less than 0 .

## A11 Proof of Proposition 10

Proof: We first get sellers' profit in this new benchmark scenario in the sell-on contract from Equation (23) with $a$ is set as 1:

$$
\begin{equation*}
\pi_{i}^{n s}=\frac{(1-\alpha)(5 \delta+4)^{2}(\delta+\theta+2)}{6\left[3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right]^{2}}, \quad \text { for } i=\{1,2,3\} \tag{25}
\end{equation*}
$$

In the sell-on contract, using the sellers' profit in new benchmark scenario (i.e., $\pi_{i}^{n s}$ in Equation (25)), in own brand scenario (i.e., $\pi_{i}^{o}$ in Equation (20)), and in new-brand scenario (i.e., $\pi_{i}^{n}$ in Equation (23)), we have the following results:
(a) $\pi_{i}^{o}>\pi_{i}^{n s}, i \in\{1,2\}$ if and only if (i) $a<a_{1}^{n s}$ or (ii) $a>a_{2}^{n s}$ Note that

$$
\pi_{i}^{o}-\pi_{i}^{n s}=\frac{(1-\alpha)(\delta+\theta+2)\left(A a^{2}+B a+C\right)}{6(a+2)\left[3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right]^{2}\left[\delta^{2}(3-\alpha)+2 \delta(\theta+6)+2(\theta+4)\right]^{2}}
$$

where $A=3 \delta^{2}\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2}>0$;
$B=2(5 \delta+4)^{2} \delta^{2} \alpha\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)-(\delta(\delta+16)+16)\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2}-(5 \delta+4)^{2} \delta^{4} \alpha^{2} ;$
$C=4(5 \delta+4)^{2} \delta^{2} \alpha\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)-2((\delta-8) \delta-8)\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2}-2(5 \delta+4)^{2} \delta^{4} \alpha^{2}$.
We can verify that this quadratic function of $A a^{2}+B a+C$ in $a$ has two real solutions (i.e., $\left.B^{2}-4 A C>0\right)$. We denote these two solutions of $a$, which leads to $A a^{2}+B a+C=0$, as $a_{1}^{n s}$ and $a_{2}^{n s}$, where $a_{1}^{n s}=\frac{-B-\sqrt{B^{2}-4 A C}}{2 A}$, and $a_{2}^{n s}=\frac{-B+\sqrt{B^{2}-4 A C}}{2 A}$. The base level demand $a$ should be non-
negative. We can easily show that $a_{1}^{n s}>0$ and $a_{2}^{n s}>0$ always hold. Therefore, $A a^{2}+B a+C>0$ if and only if (i) $a<a_{1}^{n s}$ or (ii) $a>a_{2}^{n s}$. In other words, $\pi_{i}^{o}>\pi_{i}^{n s}, i \in\{1,2\}$ if and only if (i) $a<a_{1}^{n s}$ or (ii) $a>a_{2}^{n s}$.
(b) $\pi_{i}^{n}>\pi_{i}^{n s}, i \in\{1,2\}$ if and only if (i) $a<a_{3}^{n s}$ or (ii) $a>a_{4}^{n s}$. Note that

$$
\pi_{i}^{n}-\pi_{i}^{n s}=\frac{(a-1)(1-\alpha)(\delta((3 a+2) \delta-16)-16)(\delta+\theta+2)}{6(a+2)\left[3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right]^{2}},
$$

which is positive if and only if (i) $a<1$ or (ii) $a>\frac{2\left(8 \delta+8-\delta^{2}\right)}{3 \delta^{2}}$. In other words, $\pi_{i}^{n}>\pi_{i}^{s}, i \in\{1,2\}$ if and only if (i) $a<1:=a_{3}^{n s}$ or (ii) $a>\frac{2\left(8 \delta+8-\delta^{2}\right)}{3 \delta^{2}}:=a_{4}^{n s}$.

## A12 Proof of Proposition 11

Proof: We first get sellers' profit in this new benchmark scenario in the sell-to contract from Equation (24) with $a$ is set as 1:

$$
\begin{equation*}
\pi_{i}^{n s}=\frac{(5 \delta+4)^{2}(\delta+\theta+2)}{12\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2}}, \quad \text { for } i=\{1,2,3\} \tag{26}
\end{equation*}
$$

In the sell-on contract, using the sellers' profit in new benchmark scenario (i.e., $\pi_{i}^{n s}$ in Equation (26)), in own brand scenario (i.e., $\pi_{i}^{o}$ in Equation (22)), and in new-brand scenario (i.e., $\pi_{i}^{n}$ in Equation (24)), we have the following results:
(a) $\pi_{i}^{o}>\pi_{i}^{n s}, i \in\{1,2\}$ if and only if $a<a_{5}^{n s}$. Note that

$$
\pi_{i}^{o}-\pi_{i}^{n s}=-\frac{(\delta+\theta+2)\left(a(5 \delta+4)^{2}(2 \delta+\theta+4)^{2}+2\left(\delta^{2}\left(\theta^{2}-48\right)+4 \delta^{3}(7 \theta+32)+46 \delta^{4}-8 \delta(\theta+4)(\theta+8)-8(\theta+4)^{2}\right)\right)}{12(a+2)(2 \delta+\theta+4)^{2}\left[3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right]^{2}},
$$

where $A=3 \delta^{2}\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2}>0$;
$B=2(5 \delta+4)^{2} \delta^{2} \alpha\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)-(\delta(\delta+16)+16)\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2}-(5 \delta+4)^{2} \delta^{4} \alpha^{2} ;$
$C=4(5 \delta+4)^{2} \delta^{2} \alpha\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)-2((\delta-8) \delta-8)\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2}-2(5 \delta+4)^{2} \delta^{4} \alpha^{2}$.
which is positive if and only if $a<\frac{-2 \delta^{2}\left(\theta^{2}-48\right)-8 \delta^{3}(7 \theta+32)-92 \delta^{4}+16 \delta(\theta+4)(\theta+8)+16(\theta+4)^{2}}{(5 \delta+4)^{2}(2 \delta+\theta+4)^{2}}$. In other words, $\pi_{i}^{o}>\pi_{i}^{n s}, i \in\{1,2\}$ if and only if (i) $a<\frac{-2 \delta^{2}\left(\theta^{2}-48\right)-8 \delta^{3}(7 \theta+32)-92 \delta^{4}+16 \delta(\theta+4)(\theta+8)+16(\theta+4)^{2}}{(5 \delta+4)^{2}(2 \delta+\theta+4)^{2}}:=a_{5}^{n s}$.
(b) $\pi_{i}^{n}>\pi_{i}^{n s}, i \in\{1,2\}$ if and only if (i) $a<a_{3}^{n s}$ or (ii) $a>a_{4}^{n s}$. Note that

$$
\pi_{i}^{n}-\pi_{i}^{n s}=\frac{(a-1)(\delta((3 a+2) \delta-16)-16)(\delta+\theta+2)}{12(a+2)\left[3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right]^{2}},
$$

which is positive if and only if (i) $a<1$ or (ii) $a>\frac{2\left(8 \delta+8-\delta^{2}\right)}{3 \delta^{2}}$. In other words, $\pi_{i}^{n}>\pi_{i}^{s}, i \in\{1,2\}$
if and only if (i) $a<1:=a_{3}^{n s}$ or (ii) $a>\frac{2\left(8 \delta+8-\delta^{2}\right)}{3 \delta^{2}}:=a_{4}^{n s}$.

## A13 Proof of Proposition 12

Proof: (a) We first get the platform's profit in the own brand scenario in the sell-on contract and sell-to contract from Equation (20) and (22), respectively.
$\frac{\partial \pi_{r}^{S O, o}}{\partial a}>0$ if and only if $a>a_{1}^{b l}$ because

$$
\frac{\partial \pi_{r}^{S O, o}}{\partial a}=\frac{\left(A a^{2}+B a+C\right)}{6(a+2)\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2}\left[\delta^{2}(3-\alpha)+2 \delta(\theta+6)+2(\theta+4)\right]^{2}},
$$

where $A=(\delta+1)(2 \delta+\theta+4)^{2}-\delta^{2}(\delta+2) \alpha>0$;
$B=4(\delta+1)(2 \delta+\theta+4)^{2}-4 \delta^{2}(\delta+2) \alpha$;
$C=4 \delta^{2} \alpha^{2}-4 \alpha(\delta(\delta(\delta+4)+4(\theta+3))+4(\theta+2))+4 \delta(\delta+1)(3 \delta+2 \theta+8)$.
We can verify that this quadratic function of $A a^{2}+B a+C$ in $a$ has two real solutions (i.e., $\left.B^{2}-4 A C>0\right)$. We denote these two solutions of $a$, which leads to $A a^{2}+B a+C=0$, as $a_{11}^{b l}$ and $a_{12}^{b l}$, where $a_{11}^{b l}=\frac{-B-\sqrt{B^{2}-4 A C}}{2 A}$, and $a_{12}^{b l}=\frac{-B+\sqrt{B^{2}-4 A C}}{2 A}$. The base level demand $a$ should be non-negative. We can easily show that $a_{11}^{b l}<0$ always holds. Therefore, $A a^{2}+B a+C>0$ if and only if $a>\max \left\{a_{12}^{b l}, 0\right\}$. In other words, $\frac{\partial \pi_{r}^{S o, o}}{\partial a}, i \in\{1,2\}$ if and only if (ii) $a>\max \left\{a_{12}^{b l}, 0\right\}:=a_{1}^{b l}$. $\frac{\partial \pi_{r}^{S T, o}}{\partial a}>0$ if and only if $a>a_{2}^{b l}$ because

$$
\frac{\partial \pi_{r}^{S T, o}}{\partial a}=\frac{\left(D a^{2}+E a+F\right)}{4(a+2)^{2}(3 \delta+2)(2 \delta+\theta+4)^{2}},
$$

where $D=(\delta+2)(2 \delta+\theta+4)^{2}>0$;
$E=4(\delta+2)(2 \delta+\theta+4)^{2}$;
$F=4\left(\delta^{2}(4 \theta+21)+4 \delta^{3}+\delta(\theta+4)(\theta+6)-(\theta+2)^{2}\right)$.
We can verify that this quadratic function of $D a^{2}+E a+F$ in $a$ has two real solutions (i.e., $E^{2}-4 D F>0$ ). We denote these two solutions of $a$, which leads to $D a^{2}+E a+F=0$, as $a_{21}^{b l}$ and $a_{22}^{b l}$, where $a_{21}^{b l}=\frac{-E-\sqrt{E^{2}-4 D F}}{2 D}$, and $a_{22}^{b l}=\frac{-E+\sqrt{E^{2}-4 D F}}{2 D}$. The base level demand $a$ should be non-negative. We can easily show that $a_{21}^{b l}<0$ always holds. Therefore, $D a^{2}+E a+F>0$ if and only if $a>\max \left\{a_{22}^{b l}, 0\right\}$. In other words, $\frac{\partial \pi_{r}^{S T, o}}{\partial a}$ if and only if (ii) $a>\max \left\{a_{22}^{b l}, 0\right\}:=a_{2}^{b l}$.
(b) We can get sellers' profit in the own brand scenario in the sell-on contract and sell-to contract from Equation (20) and (22), respectively. We can show that $\frac{\partial \pi_{i}^{S T, o}}{\partial a}=-\frac{\delta+\theta+2}{(a+2)^{2}(2 \delta+\theta+4)^{2}}<0$,
$i \in\{1,2\}$. Also, $\frac{\partial \pi_{i}^{S O, o}}{\partial a}=\frac{(1-\delta)(a \delta-4)[(a+4) \delta+4](\delta+\theta+2)}{2(a+2)^{2}\left[\delta^{2}(3-\delta)+2 \delta(\theta+6)+2(\theta+4)\right]^{2}}, i \in\{1,2\}$, which is positive if and only if $a>\frac{4}{8}$.

## A14 Proof of Proposition 13

Proof: We can get sellers' profit in the own brand scenario in the sell-on contract and sell-to contract from Equation (20) and (22), respectively.
(a) $\frac{\partial \pi_{i}^{S O, o}}{\partial \delta}=\frac{(1-\alpha)[(a+4) \delta+4](A a-B)}{2(a+2)\left(\delta^{2}(3-\alpha)+2 \delta(\theta+6)+2(\theta+4)\right)^{3}}, i \in\{1,2\}$, where $A=2 \delta^{2}(\theta(\alpha-2)+2 \alpha)+\delta^{3}(\alpha-3)+$ $6 \delta(\theta+4)+4(\theta+2)(\theta+4)>0 ; B=4\left(\delta\left(-\alpha\left(\delta^{2}+2(\delta+2) \theta+7 \delta+8\right)+3 \delta^{2}+4 \delta \theta+9 \delta+8 \theta+12\right)+6 \theta+8\right)$. Therefore, $\left.\frac{\partial \pi_{i}^{S O, o}}{\partial \delta}>0, i \in\{1,2\}\right)$ if and only if $a>\frac{B}{A}:=a^{c p}$.
(b) $\frac{\partial \pi_{i}^{S T, o}}{\partial \delta}=-\frac{2 \delta+3 \theta+4}{(a+2)(2 \delta+\theta+4)^{3}}, i \in\{1,2\}$, which is always negative.

## A15 Proof of Proposition 14

Proof: We first illustrate the sellers' equilibrium profit in three scenarios (i.e., scenarios $s, o$, and $n$ ) with the revenue sharing contract. In the benchmark scenario $s$, in stage 2 of the game, the platform's decision of optimal retail prices for each product is:

$$
\begin{equation*}
\max _{p_{1}^{s}, p_{2}^{s}} \pi_{r}^{s}=\left(p_{1}^{s} \gamma-w_{1}^{s}\right) D_{1}^{s}+\left(p_{2}^{s} \gamma-w_{2}^{s}\right) D_{2}^{s} \tag{27}
\end{equation*}
$$

where $D_{i}^{s}, i \in\{1,2\}$, is defined in Equation (1) and $\gamma$ is the revenue sharing portion that belongs to the platform. Solving the first-order conditions of Equation (27), we can derive the retail price of product $i, i \in\{1,2\}$ as $p_{i}^{s}=\frac{1}{2}\left(w_{i}^{s}+\gamma\right)$. In stage 1 of the game, anticipating the platform's reaction functions, the two sellers derive their optimal wholesale prices:

$$
\begin{equation*}
\max _{w_{i}^{s}} \pi_{i}^{s}=w_{i}^{s} D_{i}^{s}+(1-\gamma) p_{i}^{s} D_{i}^{s} \tag{28}
\end{equation*}
$$

Solving the sellers' optimization problem in stage 1 characterized by the first-order condition of Equation (28), we can derive the optimal wholesale price of product $i, w_{i}^{s}=\frac{2(\theta+1)}{(\theta+2)(\gamma+1)}-\frac{2(\theta+1)}{\theta+2}+\gamma$, $i \in\{1,2\}$. Substituting this optimal wholesale price into the above optimal retail price function of product $i, i \in\{1,2\}$, we derive the optimal retail price, $p_{i}^{s}=\frac{(\theta+2) \gamma+1}{(\theta+2)(\gamma+1)}$. With the equilibrium wholesale and retail prices, demand functions in Equation (1), and sellers' profit function in Equation
(28), we have equilibrium profits for the sellers

$$
\pi_{i}^{s}=\frac{\theta+1}{2(\theta+2)^{2}(\gamma+1)} .
$$

In the own brand scenario $o$, in stage 2 of the game, the platform determines optimal retail prices for each product to maximize its profit:

$$
\begin{equation*}
\max _{p_{r}^{o}, p_{1}^{o}, p_{2}^{o}} \pi_{r}^{o}=p_{r}^{o} D_{r}^{o}+\left(p_{1}^{o} \gamma-w_{1}^{o}\right) D_{1}^{o}+\left(p_{2}^{o} \gamma-w_{2}^{o}\right) D_{2}^{o} \tag{29}
\end{equation*}
$$

where $D_{r}^{o}, D_{1}^{o}$ and $D_{2}^{o}$ are specified in Equation (2) and $\gamma$ is the revenue sharing portion that belongs to the platform. Solving the platform's optimization problem in stage 2 characterized by the firstorder conditions of Equation (29), we can derive the platform's optimal retail prices as functions of the wholesale price. Then, in stage 1 of the game, the two sellers determine optimal wholesale prices for each product to maximize their perspective profits:

$$
\begin{equation*}
\max _{w_{i}^{o}} \pi_{i}^{o}=w_{i}^{o} D_{i}^{o}+(1-\gamma) p_{i}^{o} D_{i}^{o} \tag{30}
\end{equation*}
$$

The sellers' optimization problem in stage 1 is characterized by the first-order condition of Equation (30), from which we can derive the optimal wholesale price of product $i, i \in\{1,2\}$ :

$$
\begin{aligned}
w_{i}^{o} & =\frac{1}{2}\left(-\frac{(a+8) \delta+8}{3 \delta+2}-\frac{4(\delta+2)}{(\delta+\theta+2)^{2}}+\frac{3 \delta(\delta+8)+16}{(3 \delta+2)(\delta+\theta+2)}\right) \\
& +\frac{\gamma(\delta(a(\delta+\theta+2)+3 \delta+4(\theta+3))+4(\theta+2))}{2(3 \delta+2)(\delta+\theta+2)}+\frac{2(\delta+1)\left(\delta^{2}+2(\delta+1) \theta+6 \delta+4\right)}{(3 \delta+2)(\gamma+1)(\delta(\delta+\theta+6)+\theta+4)} \\
& +\frac{2(3 \delta+2) \theta\left(-\delta^{2}(\delta+\theta+2)+\delta \gamma(3 \delta(\delta+\theta+10)+12 \theta+64)+8(\theta+4) \gamma\right)}{(\delta+\theta+2)^{2}(\delta(\delta+\theta+6)+\theta+4)\left(\delta^{2} \gamma^{2}(\delta+\theta+2)+\delta^{2}(\delta+\theta+2)-2 \gamma(\delta(\delta(\delta+\theta+14)+6 \theta+32)+4(\theta+4))\right)} .
\end{aligned}
$$

With the equilibrium wholesale and retail price, demand functions in Equation (2), and sellers' profit function in Equation (30), we have equilibrium profits for the sellers

$$
\pi_{i}^{o}=\frac{\left(\delta^{2}(\gamma-1)^{2}-24 \delta \gamma-16 \gamma\right)\left(\delta^{3}(\gamma-1)^{2}+2 \delta^{2}\left(\theta(\gamma-1)^{2}+(\gamma-14) \gamma+1\right)-8 \delta(3 \theta+8) \gamma-16(\theta+2) \gamma\right)}{2(a+2)(\gamma+1)\left(\delta^{2} \gamma^{2}(\delta+\theta+2)+\delta^{2}(\delta+\theta+2)-2 \gamma(\delta(\delta(\delta+\theta+14)+6 \theta+32)+4(\theta+4))\right)^{2}} .
$$

In the new-brand scenario $n$, in stage 2 of the game, the platform determines optimal retail
prices for each product to maximize its profit:

$$
\begin{equation*}
\max _{p_{1}^{n}, p_{2}^{n}, p_{n}^{n}} \pi_{r}^{n}=\left(p_{1}^{n} \gamma-w_{1}^{n}\right) D_{1}^{n}+\left(p_{2}^{n} \gamma-w_{2}^{n}\right) D_{2}^{n}+\left(p_{n}^{n} \gamma-w_{n}^{n}\right) D_{n}^{n}, \tag{31}
\end{equation*}
$$

where $D_{n}^{n}$ and $D_{i}^{n}$ of scenario $n$ are analogous to $D_{r}^{o}$ and $D_{i}^{o}$ of scenario $o$ in Equation (2). $\gamma$ is the revenue sharing portion that belongs to the platform. Solving the platform's optimization problem in stage 2 characterized by the first-order conditions of Equation (31), we can derive the optimal retail prices as a function of the wholesale price. Then, in the stage 1, anticipating the retail prices, three sellers simultaneously determine optimal wholesale prices to maximize their own profits:

$$
\begin{align*}
\max _{w_{i}^{n}} \pi_{i}^{n} & =w_{i}^{n} D_{i}^{n}+(1-\gamma) p_{i}^{n} D_{i}^{n}, \text { for } i=\{1,2\}  \tag{32}\\
\max _{w_{n}^{n}} \pi_{n}^{n} & =w_{n}^{n} D_{n}^{n}+(1-\gamma) p_{n}^{n} D_{n}^{n},
\end{align*}
$$

Solving the sellers' optimization problem in stage 1, we can derive the optimal wholesale price of product $i, i \in\{1,2\}$, and $n$ :

$$
\begin{aligned}
& w_{i}^{n}=\frac{2(\delta+1)((a+2) \delta(3 \delta+2 \theta+6)+4(\theta+2))}{(3 \delta+2)(\gamma+1)\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)}+\frac{2(a+4) \delta+8}{3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)}+\frac{(\gamma-2)((a+2) \delta+2)}{3 \delta+2}, \quad \text { for } i=\{1,2\}, \\
& w_{n}^{n}=\frac{\gamma(a(\delta+2)+2 \delta)}{3 \delta+2}-\frac{2 \gamma\left(a \delta\left(3 \delta^{2}+2 \delta(\theta+6)+3 \theta+16\right)+2 a(\theta+4)+2 \delta(\delta+1)(3 \delta+2 \theta+6)\right)}{(3 \delta+2)(\gamma+1)\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)} .
\end{aligned}
$$

With the equilibrium wholesale and retail price, demand functions, and sellers' profit function in Equation (32), we have equilibrium profits for the seller $i, i=\{1,2\}$

$$
\pi_{i}^{n}=\frac{((a+4) \delta+4)^{2}(\delta+\theta+2)}{2(a+2)(\gamma+1)\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2}} .
$$

Using the equilibrium profits of incumbent seller $i, i=\{1,2\}$ in the benchmark scenario $s$, own brand scenario $o$, and new-brand scenario $n$ (i.e., $\pi_{i}^{s}, \pi_{i}^{o}$, and $\pi_{i}^{n}$ ), we show that
(a) $\pi_{i}^{o}>\pi_{i}^{s}, i \in\{1,2\}$ if and only if $a<a_{1}^{r s}$ and $\delta<\delta_{1}^{r s}$. Note that

$$
\pi_{i}^{o}-\pi_{i}^{s}=\frac{-A a+B}{2(a+2)(\theta+2)^{2}(\gamma+1)^{2}\left[\delta^{2} \gamma^{2}(\delta+\theta+2)+\delta^{2}(\delta+\theta+2)-2 \gamma(\delta(\delta(\delta+\theta+14)+6 \theta+32)+4(\theta+4)]^{2}\right.},
$$

where $A=(\theta+1)(\gamma+1)\left(\delta^{2} \gamma^{2}(\delta+\theta+2)+\delta^{2}(\delta+\theta+2)-2 \gamma(\delta(\delta(\delta+\theta+14)+6 \theta+32)+4(\theta+4))\right)^{2}>0$; $B=-4(\theta+1)(\gamma+1)\left(\delta^{2} \gamma^{2}(\delta+\theta+2)+\delta^{2}(\delta+\theta+2)-2 \gamma(\delta(\delta(\delta+\theta+14)+6 \theta+32)+4(\theta+4))\right)^{2}$
$+2(\theta+2)^{2}(\gamma+1)\left(\delta^{2}(\gamma-1)^{2}-24 \delta \gamma-16 \gamma\right)\left(\delta^{3}(\gamma-1)^{2}+2 \delta^{2}\left(\theta(\gamma-1)^{2}+(\gamma-14) \gamma+1\right)-8 \delta(3 \theta+8) \gamma-16(\theta+2) \gamma\right)$.
The base level demand $a$ should be non-negative. We can show that $-A a+B>0$ if and only if $a<\frac{A}{B}$, where $\frac{A}{B}>0$ if and only if $\delta<\delta_{1}^{r s}$. Therefore, $\pi_{i}^{o}<\pi_{i}^{s}$ if and only if $a<\frac{A}{B}:=a_{1}^{r s}$ and $\delta<\delta_{1}^{r s}$.
(b) $\pi_{i}^{n}>\pi_{i}^{s}, i \in\{1,2\}$ if and only if (i) $a<a_{2}^{r s}$ and $\delta<\delta_{2}^{r s}$ or (ii) $a>a_{3}^{r s}$. Note that

$$
\pi_{i}^{n}-\pi_{i}^{s}=\frac{C a^{2}+D a+E}{4(a+2)(\theta+2)^{2}(\gamma+1)\left[3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right]^{2}}
$$

where $C=\delta^{2}(\theta+2)^{2}(\gamma+1)(\delta+\theta+2)-4(\theta+1)(\gamma+1)\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2}>0$;
$D=16 \delta^{2}(\theta+2)^{2}(\gamma+1)(\delta+\theta+2)-2(\theta+1)(\gamma+1)\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2}+16 \delta(\theta+2)^{2}(\gamma+1)(\delta+\theta+2)$; $E=64 \delta(\theta+2)^{2}(\gamma+1)(\delta+\theta+2)+32(\theta+2)^{2}(\gamma+1)(\delta+\theta+2)$.

We can verify that this quadratic function of $C a^{2}+D a+E$ in $a$ has two real solutions (i.e., $\left.D^{2}-4 C E>0\right)$. We denote these two solutions of $a$, which leads to $C a^{2}+D a+E=0$, as $a_{3}^{r s}$ and $a_{2}^{r s}$, where $a_{3}^{r s}=\frac{-D+\sqrt{D^{2}-4 C E}}{2 C}$, and $a_{2}^{r s}=\frac{-D-\sqrt{D^{2}-4 C E}}{2 C}$. The base level demand $a$ should be nonnegative. We can show that $a_{3}^{r s}>0$, and $a_{2}^{r s}>0$ if and only if $\delta<\delta_{2}^{r s}$. Therefore, $C a^{2}+D a+E>0$ if and only if (i) $a<a_{2}^{r s}$ and $\delta<\delta_{2}^{r s}$ or (ii) $a>a_{3}^{r s}$, where $\delta_{2}^{r s}$ is a function of $\theta$. In other words, $\pi_{i}^{n}>\pi_{i}^{s}$ if and only if (i) $a<a_{2}^{r s}$ and $\delta<\delta_{2}^{r s}$ or (ii) $a>a_{3}^{r s}$.
(c) $\pi_{i}^{o}<\pi_{i}^{n}, i \in\{1,2\}$ because

$$
\pi_{i}^{n}-\pi_{i}^{o}=\frac{F a^{2}+G a+H}{2(a+2)(\gamma+1)\left[3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right]^{2}\left[\delta^{2} \gamma^{2}(\delta+\theta+2)+\delta^{2}(\delta+\theta+2)-2 \gamma(\delta(\delta(\delta+\theta+14)+6 \theta+32)+4(\theta+4))\right]^{2}}>0
$$

where $F=\delta^{2}(\delta+\theta+2)\left(\delta^{2} \gamma^{2}(\delta+\theta+2)+\delta^{2}(\delta+\theta+2)-2 \gamma(\delta(\delta(\delta+\theta+14)+6 \theta+32)+4(\theta+4))\right)^{2}>0$;
$G=8 \delta^{2}(\delta+\theta+2)\left(\delta^{2} \gamma^{2}(\delta+\theta+2)+\delta^{2}(\delta+\theta+2)-2 \gamma(\delta(\delta(\delta+\theta+14)+6 \theta+32)+4(\theta+4))\right)^{2}$
$+8 \delta(\delta+\theta+2)\left(\delta^{2} \gamma^{2}(\delta+\theta+2)+\delta^{2}(\delta+\theta+2)-2 \gamma(\delta(\delta(\delta+\theta+14)+6 \theta+32)+4(\theta+4))\right)^{2}>0 ;$
$H=16(\delta+1)^{2}(\delta+\theta+2)\left(\delta^{2} \gamma^{2}(\delta+\theta+2)+\delta^{2}(\delta+\theta+2)-2 \gamma(\delta(\delta(\delta+\theta+14)+6 \theta+32)+4(\theta+4))\right)^{2}$
$-\left(3 \delta^{2}+2 \delta(\theta+6)+2(\theta+4)\right)^{2}\left(\delta^{2}(\gamma-1)^{2}-24 \delta \gamma-16 \gamma\left(\delta^{3}(\gamma-1)^{2}+2 \delta^{2}\left(\theta(\gamma-1)^{2}+(\gamma-14) \gamma+1\right)\right.\right.$
$-8 \delta(3 \theta+8) \gamma-16(\theta+2) \gamma>0$.

## A16 The Spokes Model: Equilibrium Outcome and Proofs

We derive the equilibrium profits as in our baseline model, using demand functions $D_{i}^{\kappa}$ in scenario $\kappa=\{s, o, n\}$, as described in Section 6.6, where $v_{1}=v_{2}=1$, and $v_{r}=a\left(v_{3}=a\right)$ for scenario $o(n)$.

In scenario $s$, the equilibrium profit functions of products 1 and $2, \pi_{i}^{s}, i \in\{1,2\}$ and platform $\pi_{r}^{s}$, are derived as, under the sell-on contract,

$$
\begin{aligned}
& \pi_{i}^{s, S O}=\frac{3(1-\alpha)(2 h+1)^{2} t(6 g t+1)}{2(12 g t+1)^{2}} \quad \text { for } i \in\{1,2\}, \\
& \pi_{r}^{s, S O}=\frac{3 \alpha(2 h+1)^{2} t(6 g t+1)}{(12 g t+1)^{2}}
\end{aligned}
$$

and under the sell-to contract,

$$
\begin{aligned}
& \pi_{i}^{s, S T}=\frac{3(2 h+1)^{2} t(6 g t+1)}{4(12 g t+1)^{2}} \quad \text { for } i \in\{1,2\}, \\
& \pi_{r}^{s, S T}=\frac{(2 h+1)^{2}(6 g t+1)^{2}}{8 g(12 g t+1)^{2}} .
\end{aligned}
$$

In scenario $o$, the equilibrium profit functions of products 1,2 , and $r, \pi_{i}^{o}, i \in\{1,2, r\}$, are, under the sell-on contract,

$$
\begin{aligned}
& \pi_{i}^{o, S O}=\frac{(1-\alpha)(3 g t+1)\left(12 t^{2}(3 g h+g)+t(6 g+15 h+5)+1-a(6 g t+1)\right)^{2}}{3 t\left(-\alpha+72 g^{2} t^{2}+42 g t+5\right)^{2}} \quad \text { for } i \in\{1,2\}, \\
& \pi_{r}^{o, S O}=\frac{\alpha(3 g t+1)\left(12 t^{2}(3 g h+g)+t(6 g+15 h+5)+1-a(6 g t+1)\right)^{2}}{3 t\left(-\alpha+72 g^{2} t^{2}+42 g t+5\right)^{2}}+ \\
& \frac{((3 g t+1)(2 a(6 g t+1)+t(12 g(3 h t+t-1)+15 h+5)-2)-3 \alpha t(g(1-a+6 h t+2 t)+3 h+1))(a(2-\alpha+12 g t)+\alpha+12 g t(3 h t+t-1)+(2 \alpha+5)(3 h+1) t-2)}{3 t(5-\alpha+6 g t(12 g t+7))^{2}},
\end{aligned}
$$

and under the sell-to contract,

$$
\begin{aligned}
\pi_{i}^{o, S T}= & \frac{(3 g t+1)(a-2(3 h+1) t-1)^{2}}{54 t(4 g t+1)^{2}} \quad \text { for } i \in\{1,2\}, \\
\pi_{r}^{o, S T}= & \frac{(3 g t+1)(1-a+(6 h+2) t)\left(2 g t(2-a g+2 g t+g)+3 h(2 g t+1)^{2}+1\right)}{54 g t(2 g t+1)(4 g t+1)^{2}}+ \\
& \frac{(2 g(a+3 h t+t-1)+3 h+1)(2 a(6 g t+1)+t(12 g(3 h t+t-1)+15 h+5)-2)}{108 g t(2 g t+1)(4 g t+1)} .
\end{aligned}
$$

In scenario $n$, the equilibrium profit functions of products 1,2 , and $n, \pi_{i}^{n}, i \in\{1,2, r\}$, are, under the sell-on contract,

$$
\begin{aligned}
& \pi_{i}^{n, S O}=\frac{(1-\alpha)(3 g t+1)(1-a(6 g t+1)+t(6 g(6 h t+2 t+1)+15 h+5))^{2}}{3 t(6 g t+1)^{2}(12 g t+5)^{2}} \quad \text { for } i \in\{1,2\} \\
& \pi_{r}^{n, S O}=\frac{\alpha(3 g t+1)\left(2-4 a(6 g t+1)^{2}+2(6 a g t+a)^{2}+72 g^{2} t^{2}\left(2(3 h t+t)^{2}+1\right)+24 g t\left(5(3 h t+t)^{2}+1\right)+25(3 h t+t)^{2}\right)}{t(6 g t+1)^{2}(12 g t+5)^{2}}
\end{aligned}
$$

and under the sell-to contract,

$$
\begin{aligned}
& \pi_{i}^{n, S T}=\frac{(3 g t+1)(1-a(6 g t+1)+t(6 g(6 h t+2 t+1)+15 h+5))^{2}}{6 t(6 g t+1)^{2}(2 g g t+5)^{2}} \quad \text { for } i \in\{1,2\}, \\
& \pi_{r}^{n, S T}=\frac{(3 g t+1)^{2}\left(144 g^{3} t^{2}\left((1-a)^{2}+2(3 h t+t)^{2}\right)+48 g^{2} t\left((1-a)^{2}+8(3 h t+t)^{2}\right)+4(1-a)^{2} g+170 g(3 h t+t)^{2}+25(3 h+1)^{2} t\right)}{12 g t(2 g t+1)(6 g t+1)^{2}(12 g t+5)^{2}} .
\end{aligned}
$$

In our baseline model, we show the effects of platform's own brand on the incumbent sellers, and the effectiveness of different policies to protect the incumbent sellers. With the spokes model, the key messages are consistent with those in our baseline model. While the main results are qualitatively similar, we find different conditions with the newly introduced model parameters, e.g., $h, g$, and $t$. We use an index $S$ for the spokes model. First, we reproduce Propositions $1-4$.

Proposition S1 Under the sell-on contract, introduction of the platform's own brand doesn't always hurt the incumbent sellers. The incumbent sellers can be better off, i.e., $\pi_{i}^{o}>\pi_{i}^{s}, i \in\{1,2\}$, if and only if

$$
a<\hat{a}_{1} \equiv 1+\frac{(3 h+1) t(12 g t+5)}{6 g t+1}-\frac{3 t(2 h+1)(5-\alpha+6 g t(12 g t+7))}{(12 g t+1) \sqrt{2(3 g t+1)(6 g t+1)}}
$$

Proof: Under the sell-on contract, using sellers' profit in own brand scenario (i.e., $\pi_{i}^{o}$ ) and their profit in benchmark scenario (i.e., $\pi_{i}^{s}$ ) above, we can get $\pi_{i}^{o}>\pi_{i}^{s}, i \in\{1,2\}$ if and only if $a<\hat{a}_{1}$.

$$
\pi_{i}^{o}-\pi_{i}^{s}=\frac{(1-\alpha)\left(A a^{2}+B a+C\right)}{6 t(12 g t+1)^{2}\left(5-\alpha+72 g^{2} t^{2}+42 g t\right)^{2}},
$$

where $A=2(3 g t+1)(6 g t+1)^{2}(12 g t+1)^{2}>0$;

$$
\begin{aligned}
B= & -4(3 g t+1)(6 g t+1)(12 g t+1)^{2}(t(6 g((6 h+2) t+1)+15 h+5)+1)<0 ; \\
C= & -31104 g^{5}(12 h+5) t^{7}+41472 g^{4} t^{6}(g(9 h+3)-12 h-5)+36 g^{2} t^{4}\left(162 \alpha+720 g^{2}+1464 g(3 h+1)+36(18 \alpha-17) h^{2}+12(54 \alpha-173) h-763\right) \\
& +6 \mathrm{~g} \mathrm{t}^{3}\left(-9 \alpha^{2}+216 \alpha+1332 g^{2}+1524 g(3 h+1)-9\left(4 \alpha^{2}-96 \alpha+115\right) h^{2}-6\left(6 \alpha^{2}-144 \alpha+305\right) h-590\right) \\
& +\mathrm{t}^{2}\left(-9 \alpha^{2}+90 \alpha+1152 g^{2}+708 g(3 h+1)-18\left(2 \alpha^{2}-20 \alpha+25\right) h^{2}-12\left(3 \alpha^{2}-30 \alpha+50\right) h-175\right) \\
& +216 \mathrm{~g}^{3} t^{5}\left(36 \alpha+144 g^{2}+624 g(3 h+1)+36(4 \alpha-3) h^{2}+12(12 \alpha-103) h-497\right)+t(78 g+60 h+20)+2 .
\end{aligned}
$$

We can verify that this quadratic function of $A a^{2}+B a+C$ in $a$ has two real solutions (i.e., $\left.B^{2}-4 A C>0\right)$. We denote these two solutions of $a$, which leads to $A a^{2}+B a+C=0$, as $a_{11}$ and $a_{12}$, where $a_{11}=\frac{-B-\sqrt{B^{2}-4 A C}}{2 A}$, and $a_{12}=\frac{-B+\sqrt{B^{2}-4 A C}}{2 A}$. Thus, $A a^{2}+B a+C>0$ and $A>0$ if and only if (i) $a<a_{11}$ or (ii) $a>a_{12}$. However, considering the market condition, i.e., $1-\frac{(3 h+1) t(-3 \alpha+3 g t(-2 \alpha+12 g t+9)+5)}{3 g t(\alpha+12 g t+6)+2}<a<1+\frac{(3 h+1) t(12 g t+5)}{6 g t+1}$, we can easily exclude the second condition $\left(a>a_{12}\right)$. Therefore, $\pi_{i}^{o}>\pi_{i}^{s}, i \in\{1,2\}$ if and only if $a<a_{11} \equiv \hat{a}_{1}$ under the sell-on
contract.

Proposition S2 Under the sell-to contract, introduction of the platform's own brand doesn't always hurt the incumbent sellers. The incumbent sellers can be better off, i.e., $\pi_{i}^{o}>\pi_{i}^{s}, i \in\{1,2\}$, if and only if

$$
a<1+2 t+6 h t-\frac{9 t(2 h+1)(4 g t+1)}{12 g t+1} \sqrt{\frac{6 g t+1}{2(3 g t+1)}} \equiv \hat{a}_{2}
$$

Proof: Similar to the proof of Proposition S1, we can obtain the two real solutions and the following condition, for $\pi_{i}^{o}-\pi_{i}^{s}>0$, i.e., (i) $a<1+2 t+6 h t-\frac{9 t(2 h+1)(4 g t+1)}{12 g t+1} \sqrt{\frac{6 g t+1}{2(3 g t+1)}}$ or (ii) $a>1+2 t+6 h t+\frac{9 t(2 h+1)(4 g t+1)}{12 g t+1} \sqrt{\frac{6 g t+1}{2(3 g t+1)}}$. Again, considering the market condition, i.e., $1-\frac{(3 h+1) t(12 g t+5)}{12 g t+2}<a<1+2 t+6 h t$, we can also exclude the second condition. Therefore, $\pi_{i}^{o}>\pi_{i}^{s}$, $i \in\{1,2\}$ if and only if $a<1+2 t+6 h t-\frac{9 t(2 h+1)(4 g t+1)}{12 g t+1} \sqrt{\frac{6 g t+1}{2(3 g t+1)}} \equiv \hat{a}_{2}$ under the sell-to contract.

Proposition S3 Under the sell-on contract, (a) introduction of another new brand doesn't always hurt the incumbent sellers, i.e., $\pi_{i}^{n}>\pi_{i}^{s}, i \in\{1,2\}$, if and only if

$$
a<1+\frac{(3 h+1) t(12 g t+5)}{6 g t+1}-\frac{3 t(2 h+1)(12 g t+5)}{12 g t+1} \sqrt{\frac{6 g t+1}{2(3 g t+1)}} \equiv \hat{a}_{3},
$$

(b) the incumbent sellers always profit more when they compete with the platform's own brand than with another new brand, i.e., $\pi_{i}^{o}>\pi_{i}^{n}, i \in\{1,2\}$.

Proof: (a) Similar to the proof of Proposition $S 1$, we can easily prove that $\pi_{i}^{n}>\pi_{i}^{s}, i \in\{1,2\}$, if and only if $a<\hat{a}_{3}$ under the sell-on contract.
(b) Note that $\pi_{i}^{o}-\pi_{i}^{n}>0$ always holds as

$$
\pi_{i}^{o}-\pi_{i}^{n}=\frac{(1-\alpha) \alpha(3 g t+1)(10-\alpha+12 g t(12 g t+7))(1-a(6 g t+1)+t(6 g(6 h t+2 t+1)+15 h+5))^{2}}{3 t(6 g t+1)^{2}(12 g t+5)^{2}(5+6 g t(12 g t+7)-\alpha)^{2}}>0
$$

Therefore, $\pi_{i}^{o}>\pi_{i}^{n}, i \in\{1,2\}$, under the sell-on contract.

Proposition S4 Under the sell-to contract, (a) introduction of another new brand doesn't always hurt the incumbent sellers, i.e., $\pi_{i}^{n}>\pi_{i}^{s}, i \in\{1,2\}$, if and only if $a<\hat{a}_{4} \equiv \hat{a}_{3}$,
(b) the incumbent sellers always profit less when they compete with the platform's own brand than with another new brand, i.e., $\pi_{i}^{o}<\pi_{i}^{n}, i \in\{1,2\}$.

Proof: (a) Similar to the proof of Proposition S1, we can easily prove that $\pi_{i}^{n}>\pi_{i}^{s}, i \in\{1,2\}$, if and only if $a<\hat{a}_{4} \equiv \hat{a}_{3}$ under the sell-to contract.
(b) Similar to the proof of Proposition S1, we can obtain the two real solutions and the following condition, for $\pi_{i}^{o}-\pi_{i}^{n}<0$, i.e., $1-\frac{(3 h+1) t(12 g t+5)}{12 g t+2}<a<1+\frac{(3 h+1) t(12 g t+5)(24 g t+5)}{8(3 g t+1)(6 g t+1)}$. However, considering the two market conditions in scenarios $o$ and $n$, we obtain $1-\frac{(3 h+1) t(12 g t+5)}{12 g t+2}<a<$ $1+2 t+6 h t$. Because $1+2 t+6 h t<1+\frac{(3 h+1) t(12 g t+5)(24 g t+5)}{8(3 g t+1)(6 g t+1)}, \pi_{i}^{o}<\pi_{i}^{n}, i \in\{1,2\}$ always holds under the sell-to contract.

Next, we show the different effects of platform introducing the own brand (scenario o) and the new brand (scenario $n$ ) on the incumbent sellers under different contracts.

Corollary S1 $\pi_{i}^{o, S O}>\pi_{i}^{n, S O}>\pi_{i}^{n, S T}>\pi_{i}^{o, S T}$, for $i=\{1,2\}$.
Proof: From Propositions S3(b) and S4(b), we can show $\pi_{i}^{o, S O}>\pi_{i}^{n, S O}$ and $\pi_{i}^{n, S T}>\pi_{i}^{o, S T}$, $i \in\{1,2\}$. In addition, we can show that $\pi_{i}^{n, S O}>\pi_{i}^{n, S T}$ under the conditions in scenario $n$.

In the baseline model, we discuss that when the platform has an incentive to launch its own brand (Proposition 5), which pricing contracts the platform would prefer and how such preference hurts the incumbent sellers (Proposition 6). Then we consider our main questions-how the ban and its subsequent outcome resulting from the platform's best response affect the incumbent sellers (Propositions 7, 8, and 9). Using the spokes model, we show that the main findings are consistent with those from our main analyses as follows. ${ }^{\text {A1 }}$

Proposition S5 Consistent with Proposition 5, (a) the platform has an incentive to launch its own-brand (i.e., $\pi_{r}^{o, S O}>\pi_{r}^{s, S O}$ and $\pi_{r}^{o, S T}>\pi_{r}^{s, S T}$ ) when $a$ is relatively high, and (b) compared to introducing a new brand, it always prefer to introduce its own brand (i.e., $\pi_{r}^{o, S O}>\pi_{r}^{n, S O}$ and $\left.\pi_{r}^{o, S T}>\pi_{r}^{n, S T}\right)$.

Proof: (a) Under the sell-on contract, using platform's profit in own brand scenario (i.e., $\pi_{r}^{o, S O}$ ) and their profit in benchmark scenario (i.e., $\pi_{r}^{s, S O}$ ), we can get $\pi_{r}^{o, S O}>\pi_{r}^{s, S O}$ when $a$ is relatively

[^4]high.
$$
\pi_{r}^{o, S O}-\pi_{r}^{s, S O}=\frac{A a^{2}+B a+C}{3 t(12 g t+1)^{2}\left(72 g^{2} t^{2}+42 g t-\alpha+5\right)^{2}}
$$
where $A=(12 g t+1)^{2}\left(3 g t\left((6-\alpha) \alpha+72(\alpha+2) g^{2} t^{2}+48(\alpha+2) g t+20\right)+4\right)>0$;
$$
B=-(12 g t+1)^{2}\left(6 g t\left((6-\alpha) \alpha+72(\alpha+2) g^{2} t^{2}+48(\alpha+2) g t+20\right)+(3 h+1) t(-3(\alpha-9) \alpha\right.
$$
$$
+864(\alpha-1) g^{3} t^{3}+792(\alpha-1) g^{2} t^{2}-12((\alpha-21) \alpha+19) g t-20+8
$$
$$
C=t^{2}\left(6 \alpha^{2}\left(144 g^{3}(6 h(3 h+4)+7) t^{3}+12 g^{2}(6 h+5)(30 h+13) t^{2}+2 g\left(315 h^{2}+354 h+95\right) t+51 h^{2}+54 h+14\right)\right.
$$
$$
-9 \alpha(12 g t+5)\left(h^{2}(6 g t(60 g t+23)+11)+2 h(6 g t(8 g t(3 g t(12 g t+11)+13)+19)+7)+4(3 g t+1)(5 g t(2 g t(12 g t+7)+3)+1)\right)
$$
$$
-9 \alpha^{3}(2 h+1)^{2}(6 g t+1)+(3 h+1)^{2}(3 g t+1)(12 g t+1)^{2}(12 g t+5)^{2}+(12 g t+1)^{2}\left(3 g t\left(-(\alpha-6) \alpha+72(\alpha+2) g^{2} t^{2}+48(\alpha+2) g t+20\right)\right.
$$
$$
+(3 \mathrm{~h}+1) \mathrm{t}\left(-3(\alpha-9) \alpha+864(\alpha-1) g^{3} t^{3}+792(\alpha-1) g^{2} t^{2}-12((\alpha-21) \alpha+19) g t-20\right)+4
$$

When the quadratic function of $A a^{2}+B a+C$ in $a$ has two real solutions (i.e., $B^{2}-4 A C>0$ ), we have two solutions of $a$ to $A a^{2}+B a+C=0, a_{51}$ and $a_{52}$, where $a_{51}=\frac{-B-\sqrt{B^{2}-4 A C}}{2 A}$, and $a_{52}=\frac{-B+\sqrt{B^{2}-4 A C}}{2 A}$. Thus, $A a^{2}+B a+C>0$ when $a>a_{52}$, given that $A>0$. We can also verify that $a_{52}$ is within the market condition for scenarios $o$ and $s$. Therefore, $\pi_{r}^{o, S O}>\pi_{r}^{s, S O}$ when $a$ is relatively high.

Following the same steps, we can get result of $\pi_{r}^{o, S T}-\pi_{r}^{s, S T}$ and check the solutions to the associated quadratic function in $a$ and market conditions.
(b) $\pi_{r}^{o, S O}>\pi_{r}^{n, S O}$ because

$$
\pi_{r}^{o, S O}-\pi_{r}^{s, S O}=\frac{D a^{2}+E a+F}{3 t(6 g t+1)^{2}(12 g t+5)^{2}(\alpha-6 g t(12 g t+7)-5)^{2}},
$$

where $D=(6 g t+1)^{2}\left(-3 \alpha^{2}(4 g t+1)(12 g t+5)(15 g t+4)+6 \alpha(6 g t+1)(12 g t+5)^{2}(6 g t(2 g t+1)+1)\right.$

$$
\left.+4(3 g t+1)(6 g t+1)^{2}(12 g t+5)^{2}-6 \alpha^{3}(3 g t+1)\right)>0 ;
$$

$$
E=(6 g t+1)^{2}\left(-(3 h+1) t(12 g t+5)^{2}\left(-3(\alpha-9) \alpha+864(\alpha-1) g^{3} t^{3}+792(\alpha-1) g^{2} t^{2}-12((\alpha-21) \alpha+19) g t-20\right)\right.
$$

$$
+12 \alpha^{3}(3 g t+1)-6 \alpha^{2}(4 g t+1)(12 g t+5)(15 g t+4)+12 \alpha(6 g t+1)(12 g t+5)^{2}(6 g t(2 g t+1)+1)-8(3 g t+1)(6 g t+1)^{2}(12 g t+5)^{2} ;
$$

$$
F=(3 h+1) t(12 g t+5)^{2}(6 g t+1)^{2}\left(-3(\alpha-9) \alpha+864(\alpha-1) g^{3} t^{3}+792(\alpha-1) g^{2} t^{2}-12((\alpha-21) \alpha+19) g t-20\right)
$$

$$
-(3 h+1)^{2} t^{2}(12 g t+5)^{2}\left(3 \alpha^{3}(3 g t+1)-6 \alpha^{2}(6 g t+1)(g t(24 g t+19)+4)+3 \alpha(6 g t+1)^{2}(12 g t+5)(3 g t(4 g t+3)+2)\right.
$$

$$
-(3 g t+1)(6 g t+1)^{2}(12 g t+5)^{2}+(6 g t+1)^{2}\left(-6 \alpha^{3}(3 g t+1)+3 \alpha^{2}(4 g t+1)(12 g t+5)(15 g t+4)\right.
$$

$$
-6 \alpha(6 g t+1)(12 g t+5)^{2}(6 g t(2 g t+1)+1)+4(3 g t+1)(6 g t+1)^{2}(12 g t+5)^{2}
$$

For the quadratic function of $D a^{2}+E a+F$ in $a, D>0$ and $E^{2}-4 D F<0$.

$$
\pi_{r}^{o, S T}>\pi_{r}^{n, S T} \text { because }
$$

$$
\pi_{r}^{o, S T}-\pi_{r}^{s, S T}=\frac{G a^{2}+H a+I}{108 t(4 g t+1)^{2}(6 g t+1)^{2}(12 g t+5)^{2}},
$$

where $G=4(6 g t+1)^{2}(3 g t(48 g t(9 g t+7)+79)+16)>0$;
$H=4(6 g t+1)^{2}(6 g t(48 g t(t(9 g(3 h t+t-1)+30 h+10)-7)+175(3 h+1) t-79)+125(3 h+1) t-32) ;$
$I=(3 h+1) t(12 g t+5)^{2}(3 g t(24 g t(t(18 g(3 h t+t-2)+51 h+17)-22)+121(3 h+1) t-104)+34(3 h+1) t-20)+4(3 g t(48 g t(9 g t+$ 7) +79$)+16)(6 \mathrm{gt}+1)^{2}$

We can verify that this quadratic function of $G a^{2}+H a+I$ in $a$ has two real solutions (i.e., $\left.H^{2}-4 G I>0\right)$. We can also verify that the two solutions are both less than or equal to the lower bound of the feasible market condition for scenarios $o$ and $s$.

Proposition S6 As we show in Proposition 6, (a) the platform prefers the sell-on contract (i.e., $\left.\pi_{r}^{o, S O}>\pi_{r}^{o, S T}\right)$ if and only if $a<\hat{a}_{6}$ and $\alpha>\hat{\alpha}_{6}$. However, the incumbent sellers always prefer the sell-on contract (i.e., $\pi_{i}^{o, S O}>\pi_{i}^{o, S T}$ ); Moreover, (b) no matter which contract the platform prefers under the scenario $o$, its own brand introduction hurts the incumbent sellers when $a$ is relatively high.

Proof: (a) In the own brand scenario, using platform's profit under the sell-on contract (i.e., $\pi_{r}^{o, S O}$ ) and their profit under the sell-to contract (i.e., $\pi_{r}^{o, S T}$ ), we can get $\pi_{r}^{o, S O}>\pi_{r}^{o, S T}$ if and only if $a<\hat{a}_{6}$ and $\alpha>\hat{\alpha}_{6}$.

$$
\pi_{r}^{o, S O}-\pi_{r}^{o, S T}=\frac{A a^{2}+B a+C}{108 g t(2 g t+1)(4 g t+1)^{2}\left(-\alpha+72 g^{2} t^{2}+42 g t+5\right)^{2}},
$$

where $A=4 g\left(10 \alpha-\left((32 g t(3 g t+1)+1)(\alpha+3 \alpha g t)^{2}\right)+2 \alpha g t(3 g t(6 g t(72 g t(8 g t(3 g t+5)+27)+665)+709)+178)\right.$

$$
+g t(3 g t(455-12 g t(9 g t(8 g t(6 g t+5)-1)-98))+205)+11 ;
$$

$$
B=-4 g\left(20 \alpha-2 \alpha^{2}(3 g t+1)(t(g(48 g t(t(6 g((6 h+2) t+1)+33 h+11)+4)+142(3 h+1) t+35)+33 h+11)+1)\right.
$$

$$
+\alpha t(2 g(6 g t(6 g t(72 g t(4 t(g(t(6 g((6 h+2) t+1)+69 h+23)+10)+54 h+18)+27)+2116(3 h+1) t+665)+2846(3 h+1) t+709)
$$

$$
+1999(3 h+1) t+356)+193(3 h+1))+t(2 g(3 g t(4 t(3 g(9 g t(8 g t(-2 t(3 g((6 h+2) t+1)+33 h+11)-5)-134(3 h+1) t+1)
$$

$$
-494(3 h+1) t+98)-347(3 h+1))+455)-526(3 h+1) t+205)-55(3 h+1))+22
$$

$$
C=62208(4 \alpha-1) g^{7} t^{6}((6 h+2) t+1)^{2}+10368 g^{6} t^{5}((6 h+2) t+1)(40 \alpha+2(52 \alpha-17)(3 h+1) t-5)-432 g^{5} t^{4}(8(\alpha-81) \alpha
$$

$$
+4(4 \alpha(2 \alpha-279)+477)(3 h t+t)^{2}+4(8(\alpha-108) \alpha+201)(3 h+1) t-3-144 g^{4} t^{3}\left(\alpha(24 \alpha-665)+4(\alpha(36 \alpha-1595)+898)(3 h t+t)^{2}\right.
$$

$$
+4(2 \alpha(15 \alpha-529)+247)(3 h+1) t-98-12 g^{3} t^{2}\left(\alpha(99 \alpha-1418)+4(\alpha(237 \alpha-5219)+3931)(3 h t+t)^{2}\right.
$$

$$
+4(\alpha(159 \alpha-2846)+694)(3 h+1) t-455-4 g^{2} t\left(38 \alpha^{2}-356 \alpha+2(\alpha(346 \alpha-4843)+5017)(3 h t+t)^{2}\right.
$$

$$
+2(\alpha(175 \alpha-1999)+526)(3 h+1) t-205-2 g\left(2(\alpha-11)(\alpha+1)+(\alpha(143 \alpha-1538)+2315)(3 h t+t)^{2}\right.
$$

$$
+2(\alpha(22 \alpha-193)+55)(3 h+1) t-9(\alpha-5)^{2}(3 h+1)^{2} t .
$$

We can show that the quadratic function of $A a^{2}+B a+C=0$ in $a$ has two real solutions (i.e., $\left.B^{2}-4 A C>0\right), a_{61}$ and $a_{62}$, where $a_{61}=\frac{-B-\sqrt{B^{2}-4 A C}}{2 A}$, and $a_{62}=\frac{-B+\sqrt{B^{2}-4 A C}}{2 A} . A<0$ if and only if $\alpha$ is not high enough. When $A$ is negative, both $a_{61}$ and $a_{62}$ are greater than the upper bound of the feasible market condition for the scenario $o$, so $\pi_{r}^{o, S O}<\pi_{r}^{o, S T}$ if $\alpha$ is not high enough. When $A>0, a_{62}$ is greater than the upper bound of the feasible market condition but $a_{61}$ is within
the market condition when $\alpha$ is high enough. Therefore, $\pi_{r}^{o, S O}>\pi_{r}^{o, S T}$ if and only if $a<a_{61}=\hat{a}_{6}$ and $\alpha>\hat{\alpha}_{6}$, where $\hat{\alpha}_{6}$ is the threshold to ensure that $A>0$ and $a_{61}$ is within the market condition.

In the own brand scenario, using incumbent sellers' profit under the sell-on contract (i.e., $\pi_{i}^{o, S O}$ ) and their profit under the sell-to contract (i.e., $\pi_{i}^{o, S T}$ ), we can show that $\pi_{i}^{o, S O}>\pi_{i}^{o, S T}$ always holds.

$$
\pi_{i}^{o, S O}-\pi_{i}^{o, S T}=\frac{D a^{2}+E a+F}{54 t(4 g t+1)^{2}(\alpha-6 g t(12 g t+7)-5)^{2}},
$$

where $D=-\alpha^{2}-4 \alpha(3 g t(48 g t(3 g t+2)+19)+2)(6 g t+1)+(24 g t(6 g t+1)-7)(6 g t+1)^{2}$;
$E=2\left(\alpha^{2}+2(3 h+1) t\left(\alpha^{2}+\alpha(6 g t+1)(12 g t+5)(72 g t(2 g t+1)+7)-2(6 g t+1)(12 g t+5)(3 g t(12 g t+5)+2)\right)\right.$
$+4 \alpha(3 g t(48 g t(3 g t+2)+19)+2)(6 g t+1)-(6 g t+1)^{2}(24 g t(6 g t+1)-7) ;$
$F=-\alpha^{2}-4(3 h+1) t\left(\alpha^{2}+\alpha(6 g t+1)(12 g t+5)(72 g t(2 g t+1)+7)-2(6 g t+1)(12 g t+5)(3 g t(12 g t+5)+2)\right)$
$-2(3 h t+t)^{2}\left(2 \alpha^{2}+\alpha(12 g t(12 g t(12 g t+11)+37)+41)(12 g t+5)-(12 g t+5)^{2}(24 g t(3 g t+2)+7)\right)$
$-4 \alpha(3 g t(48 g t(3 g t+2)+19)+2)(6 g t+1)+(24 g t(6 g t+1)-7)(6 g t+1)^{2}$.
We can show that the quadratic function of $D a^{2}+E a+F=0$ in $a$ has two real solutions (i.e., $\left.E^{2}-4 D F>0\right), a_{63}$ and $a_{64}$, where $a_{63}=\frac{-E-\sqrt{E^{2}-4 D F}}{2 D}$, and $a_{64}=\frac{-E+\sqrt{E^{2}-4 D F}}{2 D}$. When $A$ is positive, both $a_{63}$ and $a_{64}$ are greater than the upper bound of market condition for the scenario $o$. When $A<0, a_{64}$ is greater than the upper bound of the feasible market condition and $a_{63}$ is less than the lower bound of the feasible market condition. Therefore, $\pi_{i}^{o, S O}>\pi_{i}^{o, S T}$ always holds.
(b) We can compare the thresholds obtained in Propositions S1 and S2 (i.e., $\hat{a}_{1}$ and $\hat{a}_{2}$ ) with those obtained in Propositions S5. We can verify that the thresholds described in Proposition S5 is higher than $\hat{a}_{1}$ and $\hat{a}_{2}$, meaning that when the platform prefers to introducing an own brand with a relatively high $a$, the incumbent sellers always hurt.

Proposition S7 Once the own brand is banned in the sell-on contract, the platform would introduce a new product with the sell-on contract (in this case, incumbent sellers' profit is $\pi_{i}^{n, S O}, i \in\{1,2\}$ ), introduce an own brand with the sell-to contract (in this case, incumbent sellers' profit is $\pi_{i}^{o, S T}, i \in$ $\{1,2\}$ ), or use the sell-on contract without introducing additional product (in this case, incumbent sellers' profit is $\left.\pi_{i}^{s, S O}, i \in\{1,2\}\right)$. While introducing a new product under the sell-to contract is an option, the platform would always prefer to introducing the own brand in the under the sell-to contract (see Proposition S5(b)). Consistent with Proposition 7, we can show that the ban on the platform's entry may not be effective when it is made under the sell-on contract because
such a policy can make the platform introduce its own brand under the sell-to contract, which is the worst scenario for the incumbent sellers (i.e., $\pi_{i}^{o, S T}<\pi_{i}^{o, S O}, i \in\{1,2\}$ ); Slightly different from Proposition 7, the analytical results from the spokes model show that under some market conditions, the platform would introduce a new product with the sell-on contract in response to the ban. But this difference would not change our key message. Under the market condition where $\pi_{r}^{n, S O}>\pi_{r}^{o, S T}$, the platform would have chosen to introduce an own brand under the sell-on contract if there was no policy because $\pi_{r}^{o, S O}>\pi_{r}^{n, S O}$ (see Proposition $S 5(\mathrm{~b})$ ). Thus, the policy still hurts the incumbent sellers because $\pi_{i}^{n, S O}<\pi_{i}^{o, S O}, i \in\{1,2\}$.

Proposition S8 Once the own brand is banned in the sell-to contract, the platform would introduce a new product with the sell-to contract (in this case, incumbent sellers' profit is $\pi_{i}^{n, S T}, i \in\{1,2\}$ ), introduce an own brand with the sell-on contract (in this case, incumbent sellers' profit is $\pi_{i}^{o, S O}, i \in$ $\{1,2\}$ ), or use the sell-to contract without introducing additional product (in this case, incumbent sellers' profit is $\left.\pi_{i}^{s, S T}, i \in\{1,2\}\right)$. While introducing a new product under the sell-on contract is an option, the platform would always prefer to introducing the own brand in the under the sell-on contract (see Proposition S5(b)). Consistent with Proposition 8, we can show that the ban under the sell-to contract can help the incumbent sellers because such a policy can make the platform introduce its own brand under the sell-on contract, which can help the incumbent sellers earn more profits (i.e., $\pi_{i}^{o, S T}<\pi_{i}^{o, S O}, i \in\{1,2\}$ ); Slightly different from Proposition 8, the analytical results from the spokes model show that under some market conditions, the platform would introduce a new product with the sell-to contract in response to the ban. But this difference would not change our key message. Under the market condition where $\pi_{r}^{n, S T}>\pi_{r}^{o, S O}$, the platform would have chosen to introduce an own brand under the sell-to contract if there was no policy because $\pi_{r}^{0, S T}>\pi_{r}^{n, S T}$ (see Proposition S5(b)). Thus, the policy still helps the incumbent sellers because $\pi_{i}^{n, S T}>\pi_{i}^{o, S T}$, $i \in\{1,2\}$.

Proposition S9 Once the own brand is banned in both the sell-on and sell-to contracts, the platform would introduce a new product with the sell-to contract (in this case, incumbent sellers' profit is $\pi_{i}^{n, S T}, i \in\{1,2\}$ ), introduce a new product with the sell-on contract (in this case, incumbent sellers'
profit is $\pi_{i}^{n, S O}, i \in\{1,2\}$ ), use the sell-on contract without introducing additional product (in this case, incumbent sellers' profit is $\pi_{i}^{s, S O}, i \in\{1,2\}$ ), or use the sell-to contract without introducing additional product (in this case, incumbent sellers' profit is $\pi_{i}^{s, S T}, i \in\{1,2\}$ ). Consistent with Proposition 9, we can show that while the all-out ban can sometimes help the incumbent sellers, such a policy is not as effective as the case when the ban is only under the sell-to contract. That is, the all-out ban helps the incumbent sellers earn more profit only if the platform prefers to introduce its own brand under the sell-to contract when there is no policy (i.e., $\pi_{i}^{n, S O}>\pi_{i}^{o, S T}$ and $\left.\pi_{i}^{n, S T}>\pi_{i}^{o, S T}, i \in\{1,2\}\right)$. However, the all-out ban hurts the incumbent sellers if the platform prefers to introduce its own brand under the sell-on contract when there is no policy (i.e., $\pi_{i}^{n, S O}<\pi_{i}^{o, S O}$ and $\left.\pi_{i}^{n, S T}<\pi_{i}^{o, S O}, i \in\{1,2\}\right)$.


[^0]:    ${ }^{1}$ The sell-on contract, also called the agency model, prevails in recent online marketplaces. In contrast to the traditional revenue sharing model, which is a special form of wholesale price contract (Cachon and Lariviere, 2005), upstream sellers in the agency model are granted the retail price decision and double marginalization vanishes.

[^1]:    ${ }^{2}$ It is a common assumption that the platform can procure its own product at a low price close to the marginal cost. As in the literature (e.g., Raju et al., 1995), we assume zero marginal cost. We verify that our results hold unless the marginal cost is unrealistically too significant.

[^2]:    ${ }^{3}$ We also verify that we can derive the similar results when we use a circular city model (Salop, 1979).

[^3]:    ${ }^{4}$ As is often the case in reality, there exists a segment of no-comparison consumers. For this group of consumers, the value from purchasing product $i$ is denoted as $v_{l i}$, the unit misfit cost as $t_{l}$, and the size to each product as $l$. Therefore, we derive demand of product $i$ as $D_{l i}^{\kappa}=h_{i}-g p_{i}^{\kappa}=\frac{l\left(v_{l i}-p_{i}^{\kappa}\right)}{t_{l}}$, where $h_{i}$ equals $\frac{l v_{l i}}{t_{l}}$ and $g$ equals $\frac{l}{t_{l}}$ in scenario $\kappa, \kappa \in\{s, o, n\}$.

[^4]:    ${ }^{\mathrm{A} 1}$ We assume that the size of no-comparison consumers is small, e.g., $h$ is low, and does not dominate our results.

